Contact Stress and Bending Stress Analysis of Spur Gear by Analytical Method

N. D. Narayankar, K. S. Mangrulkar

Dept. of Mechanical Engineering, N.B. Navale Sinhgad College of Engineering, Solapur. Maharashtra India.

Abstract— The Contact stress and bending stress in mating gears is the key parameter in gear design. This paper present the contact stress analysis and bending stress analysis of spur gear by Analytical method. For contact stress analysis Hertzian equation is used and for bending stress analysis Lewis equation is used. For calculating these contact stress and bending stress analysis of Spur gear both material of Pinion and Gear is made of Steel.

Keywords—Bending stress, Contact stress, Lewis equation, Hertz equation, Spur gear.

I. INTRODUCTION

Gear is a rotating cylindrical wheel having teeth cut on it and which meshes with another toothed part in order to transmit the torque or power. Gear is a critical component in the rotating machinery industry. Gears are mainly type like spur gears, bevel gears, helical gears, double helical gears, crown gears, hypoid gears, rack and pinion, worm gears, epicyclic gears etc. The application of these gears field from tiny wrist watches to huge machinery equipment such as automobile, aerospace industry, rolling, hoisting and transmitting machinery, marine engines etc.

II. LITERATURE REVIEW

Spur gear is a simplest type of gear having its teeth cut parallel to the axis of shaft on which gear is mounted. The spur gears are used to transmit power between parallel shafts. The operating efficiency of spur gear is about 98-99% (T. Shobha Rani et al 2013). They are usually employed to achieve constant drive ratio. There are several kind of stresses present in loaded and rotating gear teeth. But, out of all the stresses, root bending stress and surface contact stress calculation is the basic of stress analysis. We have to consider all the possibilities, so that the gears are proportional to keep all the stresses within design limit. Generally stresses calculated in gear design formula are not necessary true stress, can make it difficult to get correct answer on gear toothstresses, because it may not be known whether load is uniformly distributed across the face width and whether properly shared by the two or more pairs of teeth that are in mesh at the same time. Therefore, we have to make right assumption that will allow for stress concentration, residual stress, misalignment and tooth error (Sushil Kumar Tiwari et al 2012).

Theoretically, for the calculation of contact stress at the surface of mating teeth, Hertz equation is used and for determining bending stress at the root of meshing gears, Lewis formula is used. In detail study of the contact stress produced in the mating gears is the most important task in design of gears as it is the deciding parameters in finding the dimensions of the gear. Also the module of a gear plays an important role in transmitting the power between two shafts. The spur gear with higher module is the best choice for transmitting large power between the parallel shafts (Mr. Bharat Gupta et al 2012).

T. Shobha Rani et al. (T. Shobha Rani et al 2013) have used cast iron, nylon and polycarbonate as the materials of the spur gear for finite element analysis. They concluded that the deflection of cast iron is more as compared to nylon and Polycarbonate. Therefore, cast iron spur gear can be replaced with nylon gear whenever necessary to get the good efficiency, life and less noise.

Ali Raad Hassan (2009) has done a research study in which Contact stress analysis between two spur gear teeth was studied in different contact positions, representing a pair of mating gears during rotation. A platform has developed a program to plot a pair of teeth in contact. Each case was represented a sequence position of contact between these two teeth. The platform gives graphic results for the profiles of these teeth in each position and location of contact during rotation. Finite element models were made for these cases and stress analysis was done. The results were presented and finite element analysis results were compared with theoretical calculations, wherever available. Mrs. Shinde S.P. et al. have performed the bending stress analysis of spur gear in Ansys software and results are compared with the calculated theoretical values. They found that the values of stress obtained numerically were in good agreement with the theoretical results.

M. Raja, P. Phani’s work made an attempt to summarize about contact stresses developed in a mating spur gear which has involute teeth. A pair of spur gears are taken from a lathe gear box and proceeded forward to calculate contact stresses on their teeth. Contact failure in gears is currently predicted by comparing the calculated Hertz contact stress to experimentally determined allowable values for the given material. The method of calculating gear contact stress by Hertzian equation originally derived for contact between two cylinders. Analytically these contact stresses are...
calculated for different module. Vera Nikolic-Stanojevic et al. (Vera Nikolic-Stanojevic et al 2013) described the procedure which had used to determine the maximum value of active stress and stress fields on tooth flanks during the period of meshing. Finite Element Method (FEM) was used for modeling the contact of tooth flanks. From the result of equivalent stress field in contact areas, they concluded that FEM is suitable numerical method for different analysis of contact stresses on mating tooth flanks. The standard model of gear without the modification of linear tip relief profile has meshed and analyzed by Kristina Markovic et al. (Kristin Markovic et al 2011) using Finite Element Method to compare the stresses obtained from hertz theory on tooth profile.

III. CONTACT STRESS AND BENDING STRESS OF SPUR GEAR

A pair of spur gears with 20° full depth involute teeth based on the lewis equation. The velocity factor is to be used to account for dynamic load. The pinion shaft is connected to a 10Kw, 1440rpm motor. The starting torque of the motor is 150% of the rated torque. The speed reduction is 4:1. The pinion as well as the gear is made of steel ($S_u = 600 \text{ N/mm}^2$). The factor of safety can be taken as 1.5

TABLE NO.1 : MATERIAL PROPERTY OF GEAR

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Unit</th>
<th>Values (For both the gears in assembly)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>Kg/m³</td>
<td>7850</td>
</tr>
<tr>
<td>Coefficient</td>
<td>K⁻¹</td>
<td>1.2e-5</td>
</tr>
<tr>
<td>Poisson Ratio</td>
<td></td>
<td>0.3</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>MPa</td>
<td>2e5</td>
</tr>
<tr>
<td>Tensile Ultimate</td>
<td>MPa</td>
<td>600</td>
</tr>
</tbody>
</table>

$Kw = 10$, $n = 1440$ rpm, $i = 4$, $S_u = 600 \text{ N/mm}^2$, $f(S) = 1.5$, Starting torque = 150% (rated torque).

\[
Z_g = iZ_p = 4 \times 18 = 72
\]

\[
M_t = \frac{60 \times 10^6 \times (Kw)}{2\pi n_p}
\]

\[
M_t = \frac{60 \times 10^6 \times (10)}{2 \times 1440} = 66314.56 \text{ Nmm.}
\]

From Lewis form factor table we get,

\[
Y = 0.308
\]

\[
C_s = \frac{\text{Starting torque}}{\text{Rated torque}} = 1.5
\]

The velocity factor is unknown at this stage. Assuming a trial value for the pitch line velocity as 5 m/sec.

\[
C_v = \frac{3}{3 + v} = \frac{3}{3 + 5} = \frac{3}{8}
\]

It is assumed that the ratio ($\frac{\mu}{m}$) is 10.

\[
m = \left(\frac{60 \times 10^6}{\pi} \left(\frac{(Kw)C_p f(S)}{Z_p n_p C_v \left(\frac{2m}{3}\right) \left(\frac{2m}{3}\right) Y}\right)\right)^{\frac{1}{2}}
\]

where,

\[
Kw = \text{Power transmitted by gear}
\]

\[
f(S) = \text{Factor of safety}
\]

\[
Z_p = \text{Number of teeth on pinion}
\]

\[
Z_g = \text{Number of teeth on gear}
\]

\[
n_p = \text{Speed of motor}
\]

\[
C_v = \text{Coefficient of velocity}
\]

\[
b = \text{Face width of the gear tooth}
\]

\[
m = \text{Module of the gear}
\]

\[
Y = \text{lewis form factor}
\]

\[
m = \left(\frac{60 \times 10^6}{\pi} \right) \left(\frac{(10) (1.5) (1.5)}{18(1440) \left(\frac{3}{8}\right) 10 \left(\frac{600}{3}\right) (0.308)}\right)^{\frac{1}{2}}
\]

\[
m = 4.16 \text{ mm}
\]

Step 2 :- Selection of module.
The first preference value of the module is 5mm.

Trial 1.

\[
m = 5 \text{ mm}
\]

\[
\text{Diameter of Pinion (} d'_{p}\text{)} :-
\]

\[
d'_{p} = mZ_p = 5 \times 18 = 90 \text{ mm}
\]

\[
\text{Diameter of Gear (} d'_{g}\text{)} :-
\]

\[
d'_{g} = mZ_g = 5 \times 72 = 360 \text{ mm}
\]

\[
b = 10 \text{ m} = 10 \times 5 = 50 \text{ mm}
\]
check it for design, 
\[ P_c = \frac{2 \times m_t}{d_p} \frac{2 \times 66314.56}{90} = 1473.66 \text{ N} \]
\[ V = \frac{\pi \times d_p \times n_p}{60 \times 10^3} = \frac{90 \times 1440}{60 \times 10^3} = 6.7858 \text{ m/s} \]
\[ C_v = \frac{3}{3 + v} = \frac{3}{3 + 0.15} = 0.3066 \]
\[ P_{ef} = \frac{C_v \times P_t}{C_v} = \frac{1.5 \times 1473.66}{0.3066} = 7209.69 \text{ N} \]
From Lewis equation,
\[ S_b = m.b. \sigma_b Y \]
\[ S_b = 5 \times 50 \times 200 \times 0.308 \]
\[ S_b = 15400 \text{ N} \]
\[ f(S) = \frac{S_b}{P_{ef}} = \frac{15400}{7209.69} = 2.14 \]
The design is satisfactory and module should be 5 mm.
Hertzian Contact stress (\( \sigma_c \)) :-
\[ \sigma_c = \sqrt{\frac{P_c \times (\frac{1}{R_p} + \frac{1}{R_g})}{b \times C_v}} \]
Where,
\[ R_p = \text{Radius of pinion} \]
\[ R_g = \text{Radius of gear} \]
The values of youngs modulus \( E_1 \) and \( E_2 \) and Poissons ratio \( \mu \) are taken from Table no.1 which is same for pinion and gear.
\[ \sigma_c = \sqrt{\frac{1473.66 \times (1/0.25 + 1/0.37)}{50 \times (0.71 - 0.37) \times 2 \times 10^{14}}} \]
\[ \sigma_c = 169.22 \text{ N/mm}^2 \]

**IV. RESULT AND CONCLUSION**

Pitting is the surface fatigue failure which occurs due to repetition of high contact stress. The failure starts with the formation of pits which continue to grow resulting in the rupture of the tooth surface. The result found by Analytical method for contact stress of spur gear is 169.22 N/mm^2 and for bending stress of spur gear is 200N/mm^2 which is suitable for safety for the gear. Also bending stress strength of the spur gear is 15400 N which is suitable for safe gear design of bending stress so, the design is acceptable for 5mm module of the gear.

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**REFERENCES**


