Stress Concentration Minimization of 2D Simply Supported Perforated Steel Plate

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Abstract— Steel is widely used in the construction engineering industry in various combinations to construct various types of structures as flyovers, skyscrapers, plants, heavy machinery vehicle structures etc. Plates with various types of cut outs are also becoming very important due to their high applications in mainly aerospace industry and vehicle industries. These cut outs are made into plates to meet the requirement in the design of the final structures. However these cut-outs creates stress concentration and eventually reduces the mechanical strength of the structure. The present study aims at reducing this stress concentration around the central cut-out by introduction of a proposed scheme of auxiliary holes. Reduction in stress concentration with symmetric and asymmetric auxiliary holes is studied. Findings of the study are made available here as numerical data and in graphical form.

Keywords— Auxiliary hole, Cut-out plate, Finite element method, Stress Mitigation, Stress analysis

I. INTRODUCTION

Stress concentration (often called stress raisers or stress risers) is a location in an object where stress is localized. A structure is strongest when force is evenly distributed over its area, so a reduction in area, e.g., caused by a crack, or a cut out in the structure results in a localized increase in stress. A structure can fail when the concentrated stress exceeds the material's theoretical cohesive strength. Extensive literature has been published on shape optimization and stress reduction techniques for minimum stress concentration, and new methods still appear.

The stress concentration factor or theoretical stress concentration factor is defined as the ratio of the calculated peak stress to the nominal stress that would exist in the member if the distribution of stress remained uniform; that is,

$$K_t = \frac{\sigma_{max}}{\sigma_{nom}}$$

The nominal stress is found using basic strength of materials formulas, and the calculations can be based on the properties of the net cross section at the stress raiser. Sometimes the overall section is used in computing the nominal stress. The effect of the stress raiser is to changeonly the distribution of stress. Equilibrium requirements dictate that the average stress on the section be the same in the case of stress concentration as it would be if there were a uniform stress distribution. Stress concentration results not only in unusually high stresses near the stress raiser but also in unusually low stresses in the remainder of the section. [1]

Heywood [2] reported that stress concentration can be reduced by introducing smaller auxiliary holes on either side of the original hole, which smoothen the flow of the tensile principal stress trajectories past the original hole. Rajaiah et al. [3] proposed hole shape optimization for stress mitigation in a finite plate by photo elasticity method. They introduced auxiliary holes around central cut-out for mitigation of SCF and also optimized the shape of circular holes. Meguid [4] presented a technique for reduction of SCF in a uni-axially loaded plate with two coaxial holes by introducing defence hole system- material removal in the form of circular holes. Defence hole system is a technique of material removal for stress mitigation. Finite element method was used for analysis. A comprehensive plane stress finite element study of the effect of material removal upon mitigation of elastic SCF in a uni-axially loaded plate with two coaxial holes was made. Reduction in maximum SCF ranging from 7.5% to 11 % could be achieved. Giare et al. [5] presented a method for the reduction of stress concentration in an isotropic plate by using composite material rings around the hole. They have reported the reduction in stress concentration factor by reinforcement.

Kalita et al. [6] has studied the variation of deflection and induced stresses due to presence of central cut-outs under transverse loading. They have used small auxiliary holes around the central square hole to mitigate stresses in orthotropic and isotropic plates [7, 8]. The present work is an improvement of these works.

II. EQUATIONS USED

Stress analysis of an elastic body is usually three dimensional problem. But, most of the practical problems appear in the state of plane stress or plane strain. Stress analysis of three-dimensional bodies under plane stress or plane strain can be treated as two-dimensional problems. The solution of two-dimensional problems require the integration of the different equations of equilibrium together with the compatibility equations and boundary conditions. If body force is neglected, the equations to be satisfied are

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0 \quad (1)$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \sigma_{xy}}{\partial x} = 0 \quad (2)$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(\sigma_x + \sigma_y) = 0 \quad (3)$$

Substitution of stress components by displacement components u and v into Eq. (1) to (3) makes Eq. (3) redundant and Eq. (1) and (2) transforms to

$$\frac{\partial^2 u}{\partial x^2} + \frac{(1-\nu)}{2}\left(\frac{\partial^2 u}{\partial y^2}\right) + \frac{(1+\nu)}{2}\left(\frac{\partial^2 v}{\partial x\, \partial y}\right) = 0 \quad (4)$$

$$\frac{\partial^2 v}{\partial y^2} + \frac{(1-\nu)}{2}\left(\frac{\partial^2 v}{\partial x^2}\right) + \frac{(1+\nu)}{2}\left(\frac{\partial^2 u}{\partial x\, \partial y}\right) = 0 \quad (5)$$

Now we need to find u and v from a two dimensional field satisfying the two partial differential Eq. (4) and (5).Instead of determining the two functions u and v the problem can be reduced to solving a single function $\psi(x,y)$, which can be determined by satisfying Eq. (4) and (5). The displacement potential function $\psi(x,y)$ can be defined as

$$u = \frac{\partial^2 \psi}{\partial x\, \partial y} \quad (6.1)$$

$$v = -\left[(1-\nu)\left(\frac{\partial^2 \psi}{\partial y^2}\right) + 2\left(\frac{\partial^2 \psi}{\partial x^2}\right)\right] / (1-\nu) \quad (6.2)$$

By the above definitions the displacement components u and v satisfies Eq. (4) and the only condition reduced from Eq. (5) that the function $\psi(x,y)$ has to satisfy is

$$\frac{\partial^4 \psi}{\partial x^4} + 2\left(\frac{\partial^4 \psi}{\partial x^2\, \partial y^2}\right) + \frac{\partial^4 \psi}{\partial y^4} = 0 \quad (7)$$

So, now the problem is to evaluate a single function $\Psi(x,y)$ from the bi-harmonic Eq. (7), satisfying the boundary conditions specified at the boundary [9].

III. FINITE ELEMENT FORMULATION

A simply supported rectangular steel plate of 1500mm x 1000mm x 2mm (A x B x t) with a central square cut-out of side 200mm is considered for study. Material properties of the plate are taken as E = 2×10^{11} N/m^2 and Poisson's ratio 0.3. A uniformly distributed load of 1N is applied as transverse load. An 8 node shell element, (specified as SHELL 281 in ANSYS element library) with element length of 1mm near discontinuity and about 2mm at places away from the central and auxiliary hole is used throughout the study. The element has eight nodes with six degrees of freedom at each node: translations in the x, y, and z axes, and rotations about the x, y, and z-axes. Thus each element has 48 degree of freedom in total.

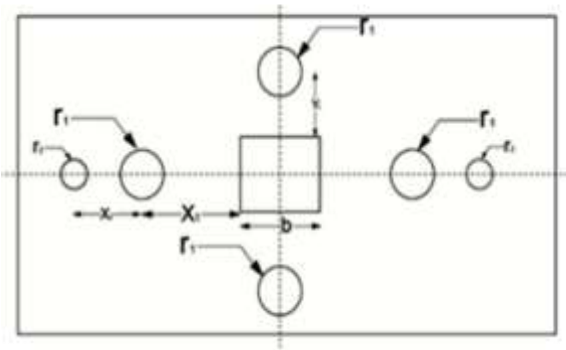

Fig. 1 Rectangular plate with central cutout and auxiliary holes.

Fig. 1 illustrates the dimensions taken. All variation of distances of auxiliary holes from the periphery of the central cut-out which are X_1, X_2, Y_1 and radius of the auxiliary holes which are r_1, r_2 are taken as functions of side of the central cut-out (i.e. b). Distance of the auxiliary holes (i.e. X_1, X_2, Y_1) is varied as distance/cut-out side length ratio (i.e. X_1/b, X_2/b, Y_1/b). Four Models of auxiliary hole placement are considered. One important concern while placing the auxiliary holes is to keep their sizes minimum.

Model 1: A single auxiliary hole with radius r_1 is placed to the left of central cut-out on y centre line at distance X_1 from the central cut-out. This Model induces asymmetry to an otherwise symmetrical plate.

Model 2: Two auxiliary holes with radius r_1 are placed on either side of the central cut-out on y centre line at distance X_1. Also the distance is kept X_1 and radius r_1 for both the holes to attain symmetry.

Model 3: Four auxiliary holes are placed, two same as Model 2 and other two with radius r_1 at x centre line at distance Y_1. Thus four symmetrical holes are placed all around the central cut-out.

Once an optimized location of X_1 and Y_1 and optimum radius r_1 is obtained, we can carry out Model 4 to check the feasibility of using a 2nd pair of auxiliary holes.

Model 4: Two more auxiliary holes with radius r_2 at a distance X_2 in the y centre line are placed around the arrangement obtained from Model 3's optimization.

IV. RESULT AND DISCUSSION

The reduction in Principal stresses by using auxiliary holes using Model 1 to Model 4 are shown in graphical form in fig.4, 6, 8, and 9. Fig.2, 3, 5, 7, 10, 11, 12 show the stress contour plots of the thin rectangular plate with and without the use of auxiliary holes.

Fig. 2 depicts the stress contour of a thin rectangular steel plate with a central square cut-out. The principal stresses generated in a solid steel plate of the given dimensions due to a transverse load of 1N is about 121451 N/m^2 and presence of an internal central cut-out raises the stresses to 352706 N/m^2 and presence of an

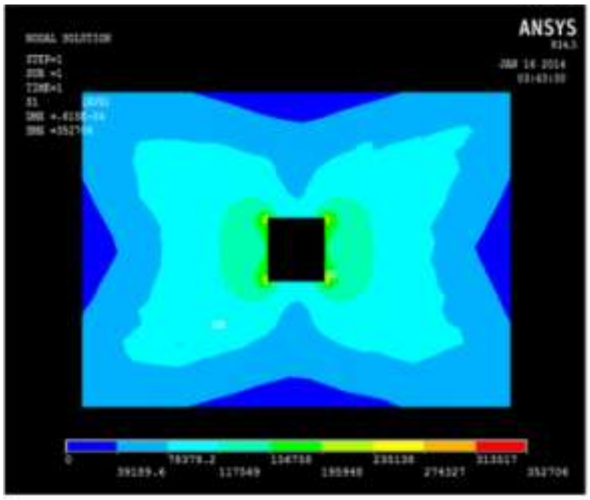

Fig. 2 Principal stress plot for a square cutout of 200 mm X 200 mm.

TABLE – I

PERCENTREDUCTION IN PRINCIPAL STRESS IN MODEL 1

Model	X_1/b	r_1/b	Principal Stresses	
			Absolute	% reduction
1	0.5	0.05	351015	0.48
	1		360062	-2.09
	1.5		347124	1.58
	0.5	0.1	355918	-0.91
	1		361086	-2.38
	1.5		345996	1.90
	0.5	0.2	354777	-0.59
	1		346613	1.73
	1.5		335941	4.75

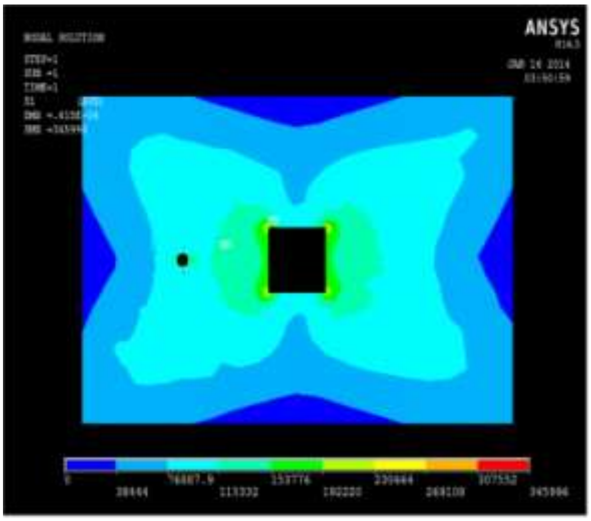

Fig. 3 Principal stress plot for model 1 with X1/b=1.5 and r1/b=0.1

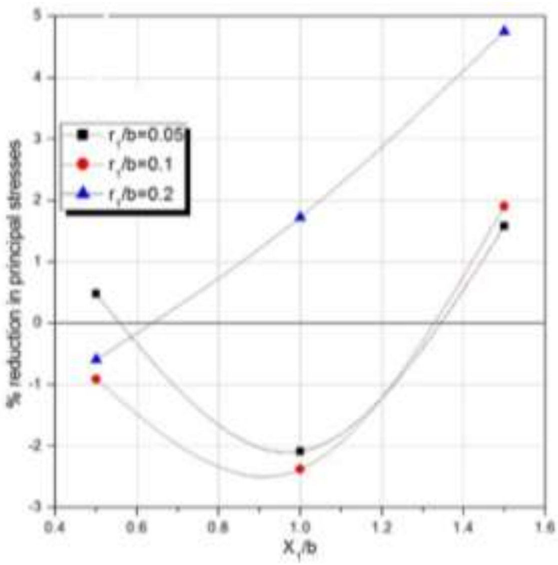

Fig. 4 Percent reduction in Principal Stress vs. X1/ b for Model 1

TABLE II

PERCENT REDUCTION IN PRINCIPAL STRESS IN MODEL 2

Model	X_1/b	r_1/b	Principal Stresses	
			Absolute	% reduction
2	0.5	0.05	348535	1.18
	1		347495	1.48
	1.5		347972	1.34
	0.5	0.1	352536	0.05
	1		349550	0.89
	1.5		351589	0.32
	0.5	0.2	338307	4.08
	1		346613	1.73
	1.5		344317	2.38

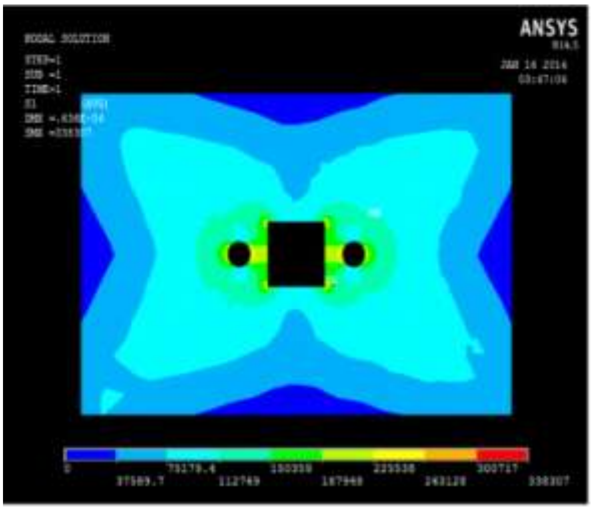

Fig. 5 Principal stress plot for model 2 with X1/b=0.5 and r1/b=0.2

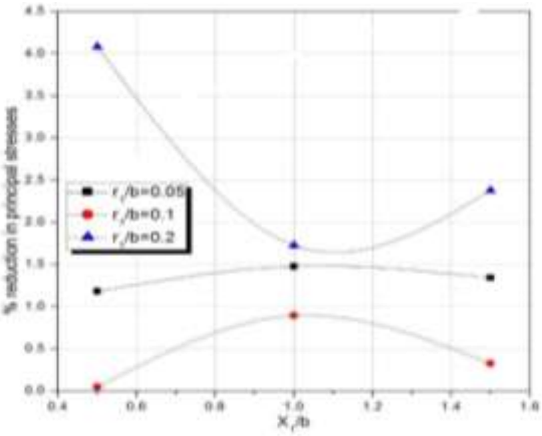

Fig. 6 Percent reduction in Principal Stress vs. X1/ b for Model 2.

the stress concentration factor is 3.01. It can be observed from fig.2 that the stresses near the corners of the square cut-out is maximum. The stresses at region near the cut-out are high but at remainder of the section it is unusually low.

Fig. 4 shows that for $r_1/b=0.05$ Model 1 shows small reduction in stress at the proximity of central cut-out for a very small radius ($r_1/b=0.05$), but as we increase the radius of the auxiliary hole (r_1) at the proximity of the central cut-out ($X_1/b=0.5$), the auxiliary hole itself becomes a stress raiser. If the r_1/b ratio is increased to 0.1 it is seen that at proximity to the central cut-out the auxiliary holes become stress raiser but at sufficient distance of $X_1/b=1.5$ a small reduction in stresses is seen. when r_1/b is increased further to 0.2, the auxiliary hole causes a rise in stress of about 0.6%, but as we go on increasing the X_1 distance the auxiliary causes better distribution of stresses and hence the stress concentration reduces to 1.73% at $X_1/b=1$ and a significant decrease of 4.75% is seen at $X_1/b=1.5$. A comparison of fig.2 and fig.3 brings out clearly that the higher stress region is less localized after the use of Model 1 as compared to fig.2, hence the reduction in stresses. Model 1 makes the plate asymmetric as only one auxiliary hole is introduced to the left of the central cut-out.

Table II contains the simulation results by using Model 2. Here the plate is symmetrical as equal radii auxiliary holes at equal distances from the boundary of the central cut-out are introduced. It is interesting to note that stress minimization is seen for all radii at all X_1 distances. This is due to the symmetrical nature of the auxiliary scheme. Fig.6 shows that the best results obtained for Model 2 is by using $r_1/b=0.2$ at a distance $X_1/b=0.5$ for which reduction in stresses is 4.08%. At a distance $X_1/b=1.5$ about 2.5 % reduction in stress is seen with $r_1/b=0.2$. The yellow region in the stress contour plot (fig. 6) shows how the localized stress (higher stress region) around the central cut-out is dispersed to wider region by the auxiliary holes on both sides of the central cut-out.

TABLE III
PERCENT REDUCTION IN PRINCIPAL STRESS IN MODEL 3

Model	X_1/b	Y_1/b	r_1/b	Principal Stresses	
				Absolute	**% reduction**
3	0.5		0.05	358833	-1.74
	1			358209	-1.56
	1.5			354514	-0.51
	0.5		0.1	359809	-2.01
	1			352927	-0.06
	1.5			353093	-0.11
	0.5		0.2	318738	9.63
	1			342456	2.91
	1.5			345317	2.09
	1.5	0.5	0.05	354895	-0.62
	1.5	1		356573	-1.10
	1.5	0.5	0.1	350848	0.53
	1.5	1		351408	0.37
	1.5	0.5	0.2	331896	5.90
	1.5	1		344494	2.33

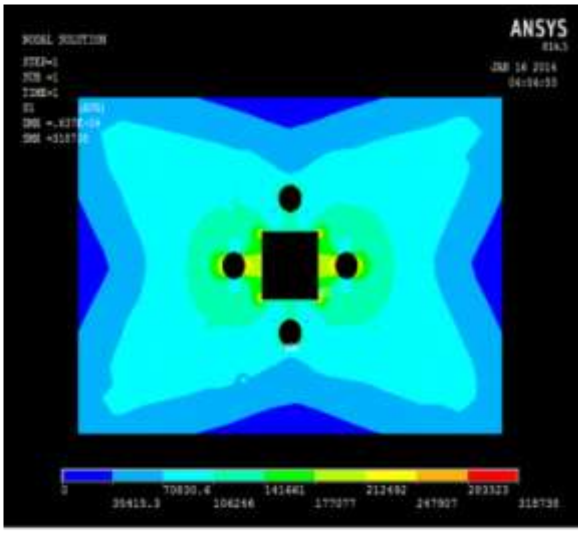

Fig. 7 Principal stress plot for model 3 with X1/b=0.5 and r1/b=0.2. Here X1=Y1

Fig.8 shows that no significant reduction in stress is seen by using Model 3 with r_1/b=0.05 and 0.1. However when Model 3 is coupled with r_1/b=0.2 and placed at close vicinity of the central cut-out it reduces stress by as much as 10%. These four auxiliary holes combined has an area of 1.3% of the plate which sufficiently meets our concern of removing minimum material to attain maximum stress reduction. Model 3 with r_1/b=0.2 has stress reduction effect for all distances but the reduction effect eases out as we move away from the central cut-out. Note that in this plots (fig.8) $X_1=Y_1$.

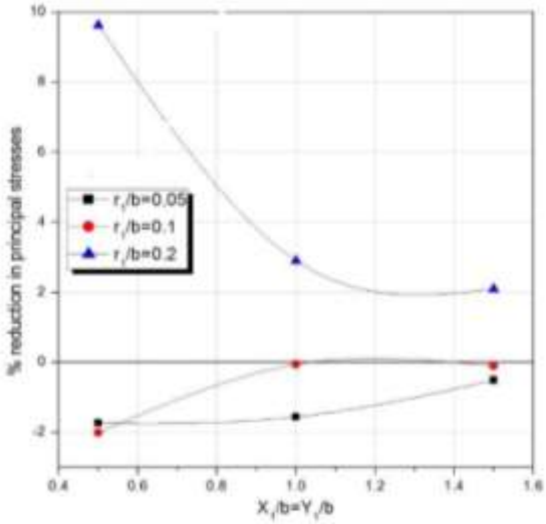

Fig. 8 Percentage reduction in Principal Stress vs. X1/ b for Model 3 (In this plots X1=Y1)

TABLE IV

PERCENT REDUCTION IN PRINCIPAL STRESS IN MODEL 4

Model	X_1/b	r_1/b	Principal Stresses	
			Absolute	% reduction
4	0.5	0.05	319875	9.31
	1		317414	10.01
	1.5		316713	10.20
	0.5	0.1	317475	9.99
	1		315218	10.63
	1.5		316353	10.31
	0.5	0.2	367182	-4.10
	1		315154	10.65
	1.5		317571	9.96

and when $X_1=Y_1$=1.5b, it means that the auxiliary holes are moving closer to the plate edge. The movement of the auxiliary holes in x-direction will not have much effect but when Y_1=1.5b the auxiliary holes are sufficiently close to the edges in y-direction it affects the stress minimization effect adversely. Data from Table III further validates this point. For ex- for r_1=0.2b, when $X_1=Y_1$=1.5b percentage reduction in stresses is 2.09 but for X_1=1.5b and Y_1=0.5b it is 5.9% and for Y_1=b it is 2.33%. However it should be noted that the auxiliary holes must be placed beyond X_1=0.5b and should not be very near the edge of the plate. Fig. 7 shows the distribution of principal stresses for $X_1=Y_1$=0.5b and r_1=0.2b plate.

Further modifications are made to improve the results obtained from Model 3. Two more auxiliary hole at distance X_2 are drilled. It is observed that at very small radius i.e. r_2/b=0.05 and 0.1 these 2nd set of auxiliary hole aid in reducing stress to about 11%. Increasing the radius r_2 has a poor effect on stress reduction at X_2< 1b

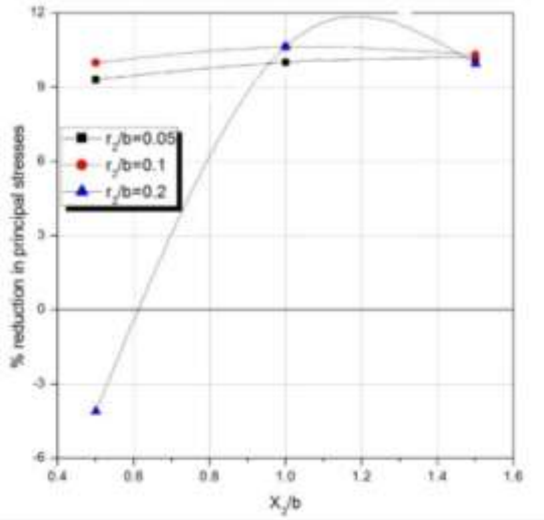

Fig. 9. Percent reduction in Principal Stress vs. X2/ b using Model 4

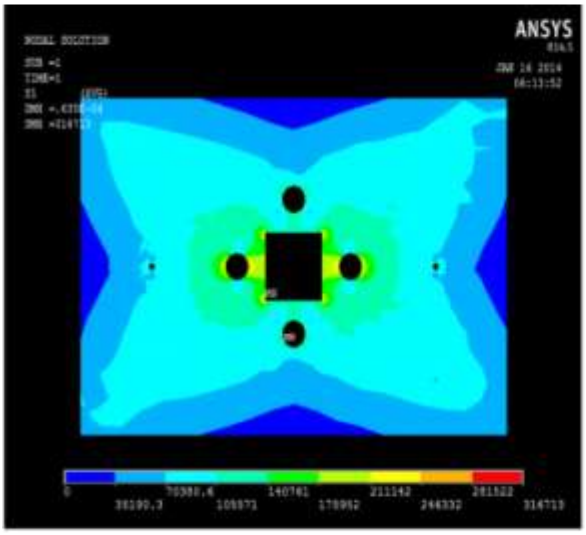

Fig. 10. Principal stress plot for model 4 with X2/b=1.5 and r2/b=0.05. (X1=Y1=100 mm, r1=40 mm)

as seen in case of r_2/b=0.2. When a comparatively larger 2nd auxiliary hole is place near the 1st set of auxiliary hole the space between this two sets of auxiliary holes (in this case denoted as X_2) will become a region of high stress concentration. Hence it should be kept in mind that if the plate dimensions are small, the use of 2^{nd}set of auxiliary holes (in this case holes with radius r_2) may not yield good results. Fig.10, 11 and 12 shows the dispersion effect of localized stresses near the central cut-out due to the presence of auxiliary holes.

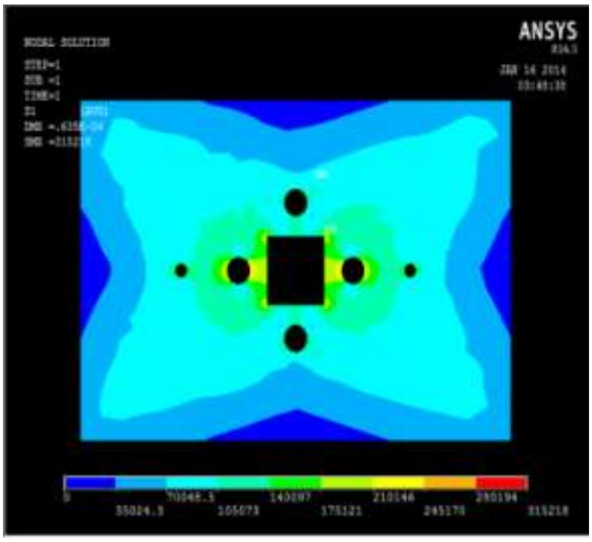

Fig. 11. Principal stress plot for model 4 with X2/b=0.1 and r2/b =0.1. (X1=Y1=100 mm, r1=40 mm)

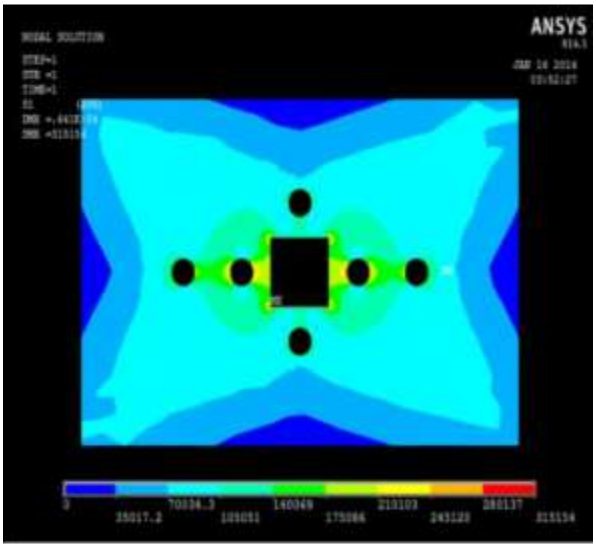

Fig. 12. Principal stress plot for model 4 with X2/b =0.1 and r2/b =0.2. (X1=Y1=100 mm, r1=40 mm)

V. CONCLUSIONS

Any abrupt change in dimensions gives rise to high stresses around the discontinuity and change in stress flow lines is seen. Through gradual change in the structure reduction in these accumulated stresses is seen. In case of plates with central cutouts this can be achieved by the proposed scheme of drilling auxiliary holes around the central cutout periphery. The distance should not be less than 0.5 times the dimension of the cutout. In general Model 2 and Model 3 seems to work better at cutout proximity of about 0.5 times the central cutout dimension. The removal of material by inclusion of auxiliary holes to reduce stress is practically more suitable for plate with infinite dimension due to sufficient availability of space and would lower the stress by significant amount. It is observed that symmetric auxiliary holes around the central cutout has better stress reduction. Also if sufficient space is available, a second smaller set of auxiliary holes will further augment the stress reduction process. By using Model 4 all the auxiliary holes combined together occupied an area of only 1.5% of the plate area for a reduction of stress of 11%.

VI. ACKNOWLEDGMENT

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