Cost-Benefit Analysis of Two Similar Warm Standby aircraft system subject to failure due to bad weather conditions and air traffic congestion; fog and wind deadliest air disasters caused by miscommunication

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Abstract : In the technological world of modern air travel, there's a certain irony in the fact that the majority of aviation disasters are caused by human error. And one of the most common forms of error is miscommunication. Even if just one person makes a mistake, the repercussions can be catastrophic.

In this paper we have taken failure due to bad weather conditions and air traffic congestion; fog and wind deadliest air disasters caused by miscommunication. When the main unit fails then warm standby system becomes operative. Failure due to fog and wind cannot occur simultaneously in both the units and after failure the unit undergoes Type-I or Type-II or Type-III or Type IV repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

Keywords: Warm Standby, failure due to bad weather conditions and air traffic congestion; fog and wind deadliest air disasters caused by miscommunication, first come first serve, MTSF, Availability, Busy period, Expected number of visits by the repairman , Benefit - Function.

INTRODUCTION

DEADLIEST AIR DISASTERS CAUSED BY MISCOMMUNICATION

Air travel is arguably one of the safest forms of transportation, but when airplane crashes do happen, because of their nature, they can take a devastating toll on human life. Here's are the worst air crashes caused by miscommunication.

Avianca Flight 52 (1990)

On January 25, 1990, Avianca Flight 52 was carrying 149 passengers from Bogotá, Colombia to New York. However, because of bad weather conditions and air traffic congestion, the Boeing 707 was forced into a holding pattern off the coast near New York. And after circling for nearly an hour and a half, the aircraft was running low on fuel.

When Flight 52 arrived at Kennedy Airport, due to the fog and wind, only one runway was open for the 33 planes that were attempting to land every hour. What's more, the flight was delayed again as the aircraft ahead of them failed to touch down. Flight 52's fuel situation soon became desperate.

Two crucial pieces of miscommunication led to the disaster that was to follow. When the aircraft was passed from regional to local air traffic controllers, the local controllers were not informed that the aircraft had too little fuel to reach its alternative airport. Compounding the problem, crucially the aircraft's crew did not explicitly declare that there was "fuel emergency" to the local controllers, which would have indicated that the plane was actually in danger of crashing.

As a result, after missing its first attempt to land, the airplane was given a landing pattern that it had too little fuel to execute. While the crew attempted to manoeuvre the plane, its engines flamed out in quick succession. The Boeing 707 slammed into the village of Cove Neck, Long Island, killing 65 of its 149 passengers and eight out of nine of its crew.

Air Florida Flight 90 (1982)

On January 13, 1982, Air Florida Flight 90 was due to travel from Washington National Airport in Virginia to Hollywood International Airport in Fort Lauderdale, Florida, with a layover in Tampa.

Conditions were snowy, and the aircraft had been de-iced improperly. Neither did it have its engine anti-icing system activated. This caused instruments to freeze and fail to register the correct readings. So, while the cabin crew thought that they had throttled up sufficiently for take-off, in actual fact they didn't have enough power.

The Boeing 737's run-up took almost half a mile (800m) longer than it should have done. Even as they set off down the runway, the first officer noticed that something was wrong with the plane's instruments and that it wasn't capable of getting airborne. However, his attempts to communicate this were brushed off by the captain, who ordered the take-off to continue.

The plane crashed into the 14th Street Bridge, killing 78 people, including four motorists. Later, reports showed that there was sufficient space for the aircraft's take-off to have been aborted - if only the flight crew had been communicating better.

Singapore Airlines Flight 006 (2000)

It's not often that an aircraft collides with a bulldozer yet, tragically, that's exactly what happened in this next accident, involving an airplane schedule to fly from Singapore to Los Angeles via Taipei. On October 31, 2000, Singapore Airlines Flight 006 was taxiing to its take-off point in stormy weather. Conditions were bad. There was low visibility thanks to the heavy rain. And crewmembers accidentally steered the Boeing 747 into runway 05R, which was closed for repairs.

The runway was cluttered with excavators, concrete barriers and a small bulldozer, but the pilot was unable to see them because of the inclement weather. The pilots had also apparently failed to read a report issued two months earlier that stated that the runway would be closed. As a result, they began take-off procedures on the wrong runway.

While attempting to take off, the aircraft collided with the heavy equipment and broke apart. Many passengers seated in the middle of the plane were killed when fuel in the wings exploded and sent fireballs through that section. The final death toll amounted to 83 of the 179 on board, including fourcrew members.

Linate Airport Disaster (2001)

On October 8, 2001, miscommunication played a role in a major collision at Linate Airport in Milan, Italy. The runway was obscured by thick fog, effectively reducing visibility to around 656 feet (200 meters), which may also have contributed to the tragedy, together with factors such as high traffic volume.

A Cessna Citation CJ2 business jet was given clearance to taxi to its take-off point on a route that would avoid the main runway. However, due partly to poor use of radio communications and lack of proper markings and signs, the Cessna misinterpreted the message and turned in the wrong direction, crossing the main runway. Its route led it into the path of Scandinavian Airlines Flight 686, a McDonnell Douglas MD-87 airliner.

The two planes collided, with Flight 686 traveling at about 170 mph (270 kph). The Cessna went up in flames, while the right engine of the MD-87 was destroyed. The pilot of Flight 686, Joakim Gustafsson, managed to get the plane airborne for a brief period. And in an attempt to regain control, he hit the thrust reverser and brakes - noted as a particularly skilful manoeuvre. Even so, Gustafsson lost control of the plane, and it smashed into a luggage hangar at the end of the runway. In total, 118 people were killed in the disaster.

Dan Air Flight 1008 (1980)

A Dan-Air 727 similar to the accident aircraft.

This disaster was caused by a single misheard word. Dan Air Flight 1008 departed from Manchester, England, on the morning of April 25, 1980, en route to Tenerife, one of Spain's Canary Islands. At 1:21 pm, the plane ploughed into the side of the island's mount La Esperanza, killing all 146 people on board.

The cause of the disaster was a misinterpretation made by the Boeing 727's flight crew. The plane was instructed by the control tower to take an unpublished, not officially approved, and potentially dangerous holding pattern above Los Rodeos Airport. But the pilot also seems to have mistaken the word "inbound" for "outbound" in the instructions he received, flying in the opposite direction to which he was supposed to.

This turn in the wrong direction took the plane through an area of exceptionally high ground. And due to the airport's lack of ground radar, the air traffic controllers were unable to tell the flight crew that the plane was off course.

Heavy clouds obscured the crew's vision, likely preventing them from seeing the looming threat of the mountain. The first sign they had of any impending danger was when the plane's ground proximity warning device was triggered. The crew attempted a steep climb, but the aircraft slammed into the mountainside, killing everyone on board instantly.

PSA Flight 182 (1978)

On September 25, 1978, Pacific Southwest Airlines Flight 182 was making a routine trip from Sacramento to San Diego. In the vicinity, an instructor was giving one of his students flying lessons in a private Cessna aircraft.

At some point, the Cessna made an unauthorized change of course, which put it on the same flight path as the much larger Boeing 727. At first, both pilots managed to steer clear of each other. But communication between the crew and airport control sounded nervous prior to the crash.

In the transmissions between air traffic control and Flight 182, the crucial word "passed" appears to have been misheard as "passing," causing the controllers to believe that the flight crew knew the location of the Cessna. In fact, they had lost sight of the plane.

Less than two minutes after the transmission, the Cessna slammed into the bottom of Flight 182's right wing. The Cessna broke to pieces, and the Boeing 727's right wing was shattered. Both aircraft plummeted into a San Diego neighbourhood, killing all 135 on-board on Flight 182, seven people on the ground, and both the Cessna pilots.

Witnesses at the crash site reported utter carnage, with blood and pieces of people's bodies strewn across the entire area. In the end, if the crew of Flight 182 had managed to clearly communicate to air traffic control that they had lost sight of the smaller plane, California's deadliest ever aircraft disaster might have been averted.

Stochastic behavior of systems operating under changing environments has widely been studied. Dhillon , B.S. and Natesan, J. (1983) studied an outdoor power systems in fluctuating environment . Kan Cheng (1985) has studied reliability analysis of a system in a randomly changing environment. Jinhua Cao (1989) has studied a man machine system operating under changing environment subject to a Markov process with two states. The change in operating conditions viz. fluctuations of voltage, corrosive atmosphere, very low gravity etc. may make a system completely inoperative. Severe environmental conditions can make the actual mission duration longer than the ideal mission duration. In this paper we have taken failure due to bad weather conditions and air traffic congestion; fog and wind deadliest air disasters caused by miscommunication. When the main unit fails then warm standby system becomes operative. Failure due to fog and wind cannot occur simultaneously in both the units and after failure the unit undergoes Type-I or repair facility of Type- II by ordinary repairman or Type III, Type IV by multispecialty repairman immediately when failure due to bad weather conditions and air traffic congestion; fog and wind deadliest air disasters caused by miscommunication e repair is done on the basis of first fail first repaired.

Assumptions

1. $\lambda_{1,}$ λ_2 λ_3 are constant failure rates when failure of warm standby, failure due to bad weather conditions and air traffic congestion; fog and wind deadliest air disasters caused by miscommunication respectively. The CDF of repair time distribution of Type I, Type II and multispecialty repairmen Type-III, IV are $G_1(t)$, $G_2(t)$ and $G_3(t)$,$G_4(t)$.
2. The failure due to fog and wind deadliest air disasters caused by miscommunication is non-

instantaneous and it cannot come simultaneously in both the units.

3. The repair starts immediately after failure due to bad weather conditions and air traffic congestion; fog and wind deadliest air disasters caused by miscommunication and works on the principle of first fail first repaired basis. The repair facility does no damage to the units and after repair units are as good as new.
4. The switches are perfect and instantaneous.
5. All random variables are mutually independent.
6. When both the units fail, we give priority to operative unit for repair.
7. Repairs are perfect and failure of a unit is detected immediately and perfectly.
8. The system is down when both the units are non-operative.

Symbols for states of the System

Superscripts O, WS, BWATF, FWF,

Operative, Warm Standby, failure due to bad weather conditions and air traffic congestion; fog and wind deadliest air disasters caused by miscommunication respectively

Subscripts nbwatf, bwatf, fwf, ur, wr, uR

No failure due to bad weather conditions and air traffic congestion deadliest air disasters caused by miscommunication; failure due to bad weather conditions and air traffic congestion deadliest air disasters caused by miscommunication, failure due to fog and wind deadliest air disasters caused by miscommunication, under repair, waiting for repair, under repair continued from previous state respectively

Up states –0, 1, 2, 3, 10 ; Down states – 4, 5, 6, 7,8,9,11

regeneration point – 0,1,2, 3, 8, 9,10

States of the System

0(O_{nbwatf}, WS_{nbwatf}) One unit is operative and the other unit is warm standby and there is no failure due to bad weather conditions and air traffic congestion caused by miscommunication of both the units.

1($BWATF_{bwatf,\ urI}$, O_{nbwatf}) The operating unit failure due to bad weather conditions and air traffic congestion caused by miscommunication is under repair immediately of Type- I and standby unit starts operating with no failure due to bad weather conditions and air traffic congestion caused by miscommunication

2($FWF_{fwf,\ urII}$, O_{nbwatf}) The operative unit failure due to fog and wind deadliest air disasters caused by miscommunication and undergoes repair of type II and the standby unit becomes operative with no failure due to bad weather conditions and air traffic congestion caused by miscommunication

3($FWF_{fwf,\ urIII}$, O_{nbwatf}) The first unit failure due to fog and wind deadliest air disasters caused by miscommunication and under Type-III multispecialty repairman and the other unit is operative with no failure due to bad weather conditions and air traffic congestion caused by miscommunication

4($BWATF_{bwatf,uR1}$, $BWATF_{bwatf,wrI}$) The unit failed due to BWATF resulting from failure due to bad weather conditions and air traffic congestion, under repair of Type- I continued from state 1and the other unit failed due to BWATF resulting from failure due to bad weather conditions and air traffic congestion caused by miscommunication is waiting for repair of Type-I.

5($BWATF_{bwatf,uR1}$, $FWF_{fwf,\ wrII}$) The unit failed due to BWATF resulting from failure of Indian satellites due to power problems in an imported component, is under repair of Type- I continued from state 1and the other unit failure due to fog and wind deadliest air disasters caused by miscommunication is waiting for repair of Type- II.

6($FWF_{fwf,\ uRII}$, $BWATF_{bwatf\ ,wrI}$) The operative unit failure due to fog and wind deadliest air disasters caused by miscommunication is under repair continues from state 2 of Type –II and the other unit failed due to BWATF resulting from failure due to bad weather conditions and air traffic congestion, is waiting under repair of Type-I.

7($FWF_{fwf\ ,uRII}$, $BWATF_{bwatf,wrII}$) The one unit failure due to fog and wind deadliest air disasters caused by miscommunication is continued to be under repair of Type II and the other unit failed due to BWATF resulting from failure due to bad weather conditions and air traffic congestion caused by miscommunication is waiting for repair of Type-II.

8($BWATF_{bwatf,urIII}$, $FWF_{fwf,\ wrII}$) The one unit failure due to bad weather conditions and air traffic congestion caused by miscommunication is under multispecialty repair of Type-III and the other unit failure due to fog and wind deadliest air disasters caused by miscommunication is waiting for repair of Type-II.

9($BWATF_{bwatf,urIII}$, $FWF_{fwf,\ wrI}$) The one unit failure due to bad weather conditions and air traffic congestion caused by miscommunication is under multispecialty repair of Type-III and the other unit failure due to fog and wind deadliest air disasters caused by miscommunication is waiting for repair of Type-I

10(O_{nbwatf} , $FWF_{fwf,\ urIV}$)

The one unit is operative with no failure due to bad weather conditions and air traffic congestion caused by miscommunication and warm standby unit failure due to fog and wind deadliest air disasters caused by miscommunication and undergoes repair of type IV.

11(O_{nbwatf} , $FWF_{fwf,\ uRIV}$)

The one unit is operative with no failure due to bad weather conditions and air traffic congestion caused by miscommunication and warm standby unit failure due to fog and wind deadliest air disasters caused by miscommunication and repair of type IV continues from state 10.

Transition Probabilities

Simple probabilistic considerations yield the following expressions:

$p_{01} = \lambda_1 / \lambda_1 + \lambda_2 + \lambda_3,$
$p_{02} = \lambda_2 / \lambda_1 + \lambda_2 + \lambda_3,$
$p_{0,10} = \lambda_3 / \lambda_1 + \lambda_2 + \lambda_3$
$p_{10} = pG_1^*(\lambda_1) + q\,G_2^*(\lambda_2),$
$p_{14} = p - pG_1^*(\lambda_1) = p_{11}^{(4)},$
$p_{15} = q - q\,G_1^*(\lambda_2) = p_{12}^{(5)},$
$p_{23} = pG_2^*(\lambda_1) + q\,G_2^*(\lambda_2),$
$p_{26} = p - pG_2^*(\lambda_1) = p_{29}^{(6)},$
$p_{27} = q - qG_2^*(\lambda_2) = p_{28}^{(7)},$
$p_{30} = p_{82} = p_{91} = 1$
$p_{0,10} = pG_4^*(\lambda_1) + q\,G_4^*(\lambda_2)$
$p_{10,1} = p - pG_4^*(\lambda_1) = p_{10,1}^{(11)}$
$p_{10,2} = q - q\,G_4^*(\lambda_2) = p_{10,2}^{(11)}$ (1)

We can easily verify that

$p_{01} + p_{02} + p_{03} = 1,$

$p_{10} + p_{14} (=p_{11}^{(4)}) + p_{15} (=p_{12}^{(5)}) = 1,$

$p_{23} + p_{26} (=p_{29}^{(6)}) + p_{27} (=p_{28}^{(7)}) = 1 p_{30} = p_{82} = p_{91} = 1$

$p_{10,0} + p_{10,1}^{(11)} (=p_{10,1}) + p_{10,2}^{(12)} (=p_{10,2}) = 1$ (2)

And mean sojourn time is

$\mu_0 = E(T) = \int_0^\infty P[T > t]\,dt$

Mean Time To System Failure

$Ø_0(t) = Q_{01}(t)[s]\, Ø_1(t) + Q_{02}(t)[s]$

$Ø_2(t) + Q_{0,10}(t)[s]\, Ø_{10}(t)$

$Ø_1(t) = Q_{10}(t)[s]\, Ø_0(t) + Q_{14}(t) + Q_{15}(t)$

$Ø_2(t) = Q_{23}(t)[s]\, Ø_3(t) + Q_{26}(t) + Q_{27}(t)$

$Ø_3(t) = Q_{30}(t)[s]\, Ø_0(t)$

$Ø_{10}(t) = Q_{10,0}(t)[s]\, Ø_{10}(t) + Q_{10,2}(t)[s]\, Ø_1(t) + Q_{10,2}(t)[s]\, Ø_2(t)$ (3-6)

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-7) and solving for

$ø_0^*(s) = N_1(s) / D_1(s)$ (7)

where

$N_1(s) = \{Q_{01}^* + Q_{0,10}^*\, Q_{10,1}^*\}\, [\, Q_{14}^*(s) + Q_{15}^*(s)\,] + \{Q_{02}^* + Q_{0,10}^*\, Q_{10,2}^*\}\, [\, Q_{26}^*(s) + Q_{27}^*(s)\,]$

$D_1(s) = 1 - \{Q_{01}^* + Q_{0,10}^*\, Q_{10,1}^*\}\, Q_{10}^* - \{Q_{02}^* + Q_{0,10}^*\, Q_{10,2}^*\}\, Q_{23}^*\, Q_{30}^* - Q_{0,10}^*\, Q_{10,0}^*$

Making use of relations (1) & (2) it can be shown that $ø_0^*(0) = 1$, which implies that $ø_0(t)$ is a proper distribution.

$$\text{MTSF} = E[T] = \left.\frac{d}{ds}\, ø_0^*(s)\right|_{s=0}$$

$$= (D_1'(0) - N_1'(0)) \,/\, D_1(0)$$

$= (\mu_0 + \mu_1 (p_{01} + p_{0,10}\, p_{10,1}) + (p_{02} + p_{0,10}\, p_{10,2})(\mu_2 + \mu_3) + \mu_{10}\, p_{0,10} \,/\, (1 - (p_{01} + p_{0,10}\, p_{10,1})\, p_{10} - (p_{02} + p_{0,10}\, p_{10,2})\, p_{23}) - p_{0,10}\, p_{10,0}$

where

$\mu_0 = \mu_{01} + \mu_{02} + \mu_{0,10},$

$\mu_1 = \mu_{10} + \mu_{11}^{(4)} + \mu_{12}^{(5)},$

$\mu_2 = \mu_{23} + \mu_{28}^{(7)} + \mu_{29}^{(6)},$

$\mu_{10} = \mu_{10,0} + \mu_{10,1} + \mu_{10,2}$

Availability analysis

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have

$M_0(t) = e^{-\lambda_1 t}\, e^{-\lambda_2 t}\, e^{-\lambda_3 t}$

, $M_1(t) = p\, \overline{G_1(t)}\, e^{-\lambda_1 t}$

$M_2(t) = q\, \overline{G_2(t)}\, e^{-\lambda_2 t},$

$M_3(t) = \overline{G_3(t)}, M_{10}(t) = \overline{G_4(t)}\, e^{-\lambda_3 t}$

The point wise availability $A_i(t)$ have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t) + q_{0,10}(t)[c]A_{10}(t)$

$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{12}^{(5)}(t)[c]A_2(t) + q_{11}^{(4)}(t)[c]A_1(t),$

$A_2(t) = M_2(t) + q_{23}(t)[c]A_3(t) + q_{28}^{(7)}(t)[c]\, A_8(t) + q_{29}^{(6)}(t)]\, [c]A_9(t)$ $A_3(t) = M_3(t) + q_{30}(t)[c]A_0(t)$

$A_8(t) = q_{82}(t)[c]A_2(t)$

$A_9(t) = q_{91}(t)[c]A_1(t)$

$A_{10}(t) = M_{10}(t) + q_{10,0}(t)[c]A_0(t) + q_{10,1}^{(11)}(t)[c]A_1(t) + q_{10,2}^{(11)}(t)[c]A_2(t)$ (8-15)

Taking Laplace Transform of eq. (8-15) and solving for $\hat{A}_0(s)$

$\hat{A}_0(s) = N_2(s) / D_2(s)$ (16)

where

$N_2(s) = \{\hat{q}_{0,10}\, \widetilde{M}_{10} + \widetilde{M}_0\}\, [\{1 - \hat{q}_{11}^{(4)}\}\{1 - \hat{q}_{28}^{(7}\, \hat{q}_{82}\} - \hat{q}_{12}^{(5)}\, \hat{q}_{29}^{(6)}\, \hat{q}_{91}] + \{\hat{q}_{01} + \hat{q}_{0,10}\, \hat{q}_{10,1}^{(11)}\}[\widetilde{M}_1\{1 - \hat{q}_{28}^{(7)}\, \hat{q}_{82}\} + \hat{q}_{12}^{(5)}\, \hat{q}_{23}\, \widetilde{M}_3 + \widetilde{M}_2] + \{\hat{q}_{02} + \hat{q}_{0,10}\, \hat{q}_{10,2}^{(11)}\}\, [\{\hat{q}_{23}\, \widetilde{M}_3\}\{1 - \hat{q}_{11}^{(4)}\} + \hat{q}_{29}^{(6)}\, \hat{q}_{91}\, \widetilde{M}_1]$

$D_2(s) = \{1 - \hat{q}_{11}^{(4)}\}\{1- \hat{q}_{28}^{(7} \hat{q}_{82}\}- \hat{q}_{12}^{(5)} \hat{q}_{29}^{(6)} \hat{q}_{91} - \{ \hat{q}_{01}+ \hat{q}_{0,10} \hat{q}_{10,1}^{(11)} \}[\hat{q}_{10} \{1 - \hat{q}_{28}^{(7)} \hat{q}_{82}\} + \hat{q}_{12}^{(5)} \hat{q}_{23} \hat{q}_{30}] - \{ \hat{q}_{02} + \hat{q}_{0,10} \hat{q}_{10,2}^{(11)}\}\{[\hat{q}_{23} \hat{q}_{30} \{1 - \hat{q}_{11}^{(4)}\}+ \hat{q}_{29}^{(6)} \hat{q}_{91} \hat{q}_{10}]$

(Omitting the arguments s for brevity)

The steady state availability

$A_0 = \lim_{t\to\infty}[A_0(t)]$

$= \lim_{s\to 0}[s\,\hat{A}_0(s)] \quad = \lim_{s\to 0} \frac{s\,N_2(s)}{D_2(s)}$

Using L' Hospitals rule, we get

$$A_0 = \lim_{s\to 0} \frac{N_2(s)+s\,N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)} \quad (17)$$

Where

$N_2(0) =\{p_{0,10} \tilde{M}_{10}(0)+ \tilde{M}_0(0) \} [\{1 - p_{11}^{(4)}\}\{1- p_{28}^{(7)} \}- p_{12}^{(5)} p_{29}^{(6)}] + \{ p_{01}+ p_{0,10} p_{10,1}^{(11)}\}[\tilde{M}_1(0)\{1 - p_{28}^{(7)} \} +p_{12}^{(5)} p_{23} \tilde{M}_3(0)+\tilde{M}_2(0)]+\{ p_{02} +p_{0,10} p_{10,2}^{(11)}\} [\{p_{23} \tilde{M}_3(0)+ \tilde{M}_2(0) \}\{1 -p_{11}^{(4)}\}+ p_{29}^{(6)} \tilde{M}_1(0)]$

$D_2'(0) =\mu_0[p_{10} (1- p_{28}^{(7)} \}+ p_{12}^{(5)} p_{23}]+ \mu_1[p_{29}^{(6)} + p_{01} p_{23} - p_{0,10} \{p_{10,0}\{1- p_{28}^{(7)} \}+p_{23} p_{10,2}^{(11)} p_{23}\}]+ \mu_2[(1-p_{11}^{(4)}) - p_{01} p_{10} -p_{0,10} (p_{10} - p_{10} p_{10,2}^{(11)} + p_{12}^{(5)} p_{10,0})] \} + \mu_3 [p_{23}[p_{12}^{(5)}\{p_{01} + p_{0,10} p_{10,1}^{(11)}\}+(1 - p_{11}^{(4)}\}\{ p_{02} + p_{0,10} p_{10,2}^{(11)} \}]+ \mu_8 [p_{28}^{(7)}(1- p_{0,10} p_{10,0} - p_{10}\{ p_{01}+ p_{0,10} p_{10,1}^{(11)}\})] + \mu_9 [p_{29}^{(6)}\{ p_{12}^{(5)} (1- p_{0,10} p_{10,0} +(p_{02} + p_{0,10} p_{10,2}^{(11)}\})] + \mu_{10} [p_{29}^{(6)}\{ p_{12}^{(5)} (1- p_{0,10} p_{10,0} +(p_{02} + p_{0,10} p_{10,2}^{(11)}\})]$

and

$\mu_3 = \mu_{30}$, $\mu_9 = \mu_{91}$, $\mu_8 = \mu_{81}$

The expected up time of the system in (0,t] is

$\lambda_u(t) = \int_0^{\infty} A_0(z)\,dz$

So that $\bar{\lambda}_u(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(S)}{SD_2(S)}$ (18)

The expected down time of the system in (0,t] is

$\lambda_d(t) = t- \lambda_u(t)$

So that $\bar{\lambda}_d(s) = \frac{1}{s^2} - \bar{\lambda}_u(s)$ (19)

The expected busy period of the server when there is failure due to bad weather conditions and air traffic congestion caused by miscommunication and failure due to fog and wind deadliest air disasters caused by miscommunication in (0,t]-R_0

$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t) + q_{0,10}(t)[c]R_{10}(t)$

$R_1(t) = S_1(t) + q_{10}(t)[c]R_0(t) +$
$\quad q_{12}^{(5)}(t)[c]\, R_2(t) + q_{11}^{(4)}(t)[c]R_1(t)$

$R_2(t) = S_2(t) + q_{23}(t)[c]R_3(t) + q_{28}^{(7)}(t)$
$\quad R_8(t) +q_{29}^{(6)}(t)][c]R_9(t)$

$R_3(t) = S_3(t) + q_{30}(t)[c]R_0(t)$

$R_8(t) = S_8(t) + q_{82}(t)[c]R_2(t)$

$R_9(t) = S_9(t) + q_{91}(t)[c]R_1(t)$

$R_{10}(t) = S_{10}(t) + q_{10,0}(t)[c]R_0(t) +q_{10,1}^{(11)}(t)[c]R_1(t)+ q_{10,2}^{(11)}(t)[c]R_2(t)$ (20-26)

where

$S_1(t) =p\, G_1\overline{(t)}\, e^{-\lambda_1 t}$,

$S_2(t) =q\, G_2\overline{(t)}\, e^{-\lambda_2 t}$

$S_3(t) = S_8(t)= S_9(t) = G_3(t)$ —

$S_{10}(t) = G_4\overline{(t)}$ (27)

Taking Laplacė Transform of eq. (20-26) and solving for $\bar{R}_0(s)$

$\bar{R}_0(s) = N_3(s) / D_2(s)$ (28)

where

$N_3(s) =\{ \hat{q}_{01} + \hat{q}_{0,10} \hat{q}_{10,1}^{(11)} \}[\hat{S}_1(1 - \hat{q}_{28}^{(7)} \hat{q}_{82}\} + \hat{q}_{12}^{(5)}[\hat{S}_2 + \hat{q}_{23} \hat{S}3+ \hat{q}_{28}^{(7)} \hat{S}8+ \hat{q}_{29}^{(6)} \hat{S}9)]]+ \{ \hat{q}_{02} + \hat{q}_{0,10} \hat{q}_{10,2}^{(11)} \} [\{ \hat{S}_2+ \hat{q}_{23}\hat{S}_3 + \hat{q}_{28}^{(7)} \hat{S}_8 + \hat{S}_9 \hat{q}_{29}^{(6)})(1- \hat{q}_{11}^{(4)})+ \hat{S}_1 \hat{q}_{29}^{(6)} \hat{q}_{91}] + \hat{q}_{0,10} \hat{S}_{10} [\{1- \hat{q}_{28}^{(7)} \hat{q}_{82} \}\{1- \hat{q}_{11}^{(4)}\}- \hat{q}_{29}^{(6)} \hat{q}_{91} \hat{q}_{12}^{(5)}]$

and $D_2(s)$ is already defined.

(Omitting the arguments s for brevity)

In the long run, $R_0 = \frac{N_3(0)}{D_2'(0)}$ (29)

Where

$N_3(0) =\{p_{01} +p_{0,10} p_{10,1}^{(11)} \}[\hat{S}_1(1 - p_{28}^{(7)} \} +p_{12}^{(5)}[\hat{S}_2 +p_{23} \hat{S}3+p_{28}^{(7)} \hat{S}8+p_{29}^{(6)} \hat{S}9)]]+ \{p_{02} +p_{0,10} p_{10,2}^{(11)} \} [\{ \hat{S}_2+ p_{23}\hat{S}_3 +p_{28}^{(7)} \hat{S}_8 + \hat{S}_9 p_{29}^{(6)})(1- p_{11}^{(4)})+ \hat{S}_1 p_{29}^{(6)}] + p_{0,10} \hat{S}_{10} [\{1-p_{28}^{(7)} \}\{1- p_{11}^{(4)}\}- p_{29}^{(6)} p_{12}^{(5)}]$

and $D_2'(0)$ is already defined.

The expected busy period of the server when there is failure due to bad weather conditions and air traffic congestion caused by miscommunication and failure due to fog and wind deadliest air disasters caused by miscommunication in (0,t] is

$\lambda_{rv}(t) = \int_0^{\infty} R_0(z)\,dz$

So that $Q_{01}{}^{*}\, \bar{\lambda}_{rv}(s) = \frac{\hat{R}_0(s)}{s}$

The expected number of visits by the repairman Type-I or Type-II for repairing the identical units in (0,t]-H_0

$H_0(t) = Q_{01}(t)[s][1+ H_1(t)] + Q_{02}(t)[s][1+H_2(t)]+Q_{0,10}(t)[s]\, H_{10}(t)]$

$H_1(t) = Q_{10}(t)[s]H_0(t)] + Q_{12}^{(5)}(t)[s]$
$\quad H_8(t) + Q_{11}^{(4)}(t)] [s]H_1(t)$,

$H_2(t) = Q_{23}(t)[s]H_3(t) + Q_{28}^{(7)}(t)\ [s]$

$H_8(t) + Q_{29}^{(6)}(t)]\ [c]H_9(t)$

$H_3(t) = Q_{30}(t)[s]H_0(t)$

$H_8(t) = Q_{82}(t)[s]H_2(t)$

$H_9(t) = Q_{91}(t)[s]H_1(t)$

$H_{10}(t) = Q_{10,0}(t)[s]H_{10}(t)] + Q_{10,1}^{(11)}(t)[s]H_1(t)] + Q_{10,2}^{(11)}(t)[s]\ H_2(t)]$ (30-35)

Taking Laplace Transform of eq. (30-35) and solving for $H_0^*(s)$

$H_0^*(s) = N_4(s) / D_3(s)$ (36)

$N_4(s) = \{ Q_{01}^* + Q_{02}^*\}[\ \{ 1 - Q_{11}^{(4)*}\}\{1-Q_{28}^{(7)*}\ Q_{82}^*\ \} - Q_{12}^{(5)*}\ Q_{29}^{(6)*}\ Q_{91}^*\ ]$

And

$D_3(s) = \{1 - Q_{11}^{(4)*}\}\ \{\ 1- Q_{28}^{(7)*}\ Q_{82}^*\} - Q_{12}^{(5)*}\ Q_{29}^{(6)*}\ Q_{91}^*](1- Q_{0,10}^*\ Q_{10,0}^*)-\{\ Q_{01}^* + Q_{0,10}^*\ Q_{10,1}^{(11)*}\}[\ Q_{10}^*\{\ 1 - Q_{28}^{(7)*}\ Q_{82}^*\ \} + Q_{12}^{(5)*}\ Q_{23}^*\ Q_{30}^*] - \{Q_{02}^* + Q_{0,10}^*\ Q_{10,2}^{(11)*}\}[\ Q_{23}^*\ Q_{30}^*\{1 - Q_{11}^{(4)*}\} + Q_{29}^{(6)*}\ Q_{91}^*\ Q_{10}^*]$

(Omitting the arguments s for brevity)

In the long run,

$H_0 = N_4(0) / D_3'(0)$ (37)

where

$N_4(0) = \{1 - p_{0,10}\}[\ \{1 - p_{11}^{(4)}\}\ \{\ 1- p_{28}^{(7)}\ \} - p_{12}^{(5)}\ p_{29}^{(6)}]$

The expected number of visits by the multispecialty repairman Type-III for repairing the identical units in (0,t]-W_0

$W_0(t)=Q_{01}(t)[s]W_1(t)+ Q_{02}(t)[s]\ W_2(t) + Q_{10,0}(t)[s]\ W_{10}(t)$

$W_1(t) = Q_{10}(t)[s]W_0(t)] + Q_{12}^{(5)}(t)[s]$

$W_2(t) + Q_{11}^{(4)}(t)]\ [s]W_1(t)$,

$W_2(t) = Q_{23}(t)[s]W_3(t) + Q_{28}^{(7)}(t)\ [s]$

$W_8(t) + Q_{29}^{(6)}(t)]\ [c]W_9(t)$

$W_3(t) = Q_{30}(t)[s][1+W_0(t)\ ]$

$W_8(t) = Q_{82}(t)[s][1+W_2(t)\ ]$

$W_9(t) = Q_{91}(t)[s][1+W_1(t)\ ]$

$W_{10}(t)=Q_{10,0}(t)[s]W_0(t)+ Q_{10,1}^{(11)}(t)[s]\ W_1(t) + Q_{10,2}^{(12)}(t)[s]\ W_2(t)$ (38-44)

Taking Laplace Transform of eq. (38-44) and solving for $H_0^*(s)$

$H_0^*(s) = N_5(s) / D_3(s)$ (45)

$N_5(s) = \{Q_{01}^* + Q_{0,10}^*Q_{0,10}^{(11)*}\ \}[Q_{12}^{(5)*}\ [\ Q_{23}^*\ Q_{30}^* + Q_{28}^{(5)*}\ Q_{82}^* + Q_{29}^{(6)*}\ Q_{91}^*\ ] + \{Q_{02}^* + Q_{0,10}^*Q_{10,2}^{(11)*}\}[\ [\ Q_{23}^*\ Q_{30}^* + Q_{28}^{(5)*}\ Q_{82}^* + Q_{29}^{(6)*}\ Q_{91}^*\ \{1 - Q_{11}^{(4)*}\}]$

(Omitting the arguments s for brevity)

In the long run,

$W_0 = N_5(0) / D_3'(0)$ (46)

where $N_5(0) = \{p_{01}+ p_{0,10}\ p_{10,1}^{(11)}\ \}$

$p_{12}^{(5)} + \{\ p_{02}+ p_{0,10}\ p_{10,2}^{(11}\}\ \{1 - p_{11}^{(4)}\}]$

The expected number of visits by the multispecialty repairman Type-III for repairing the identical units in (0,t]-Y_0

$Y_0(t)=Q_{01}(t)[s]Y_1(t)+ Q_{02}(t)[s]$

$Y_2(t) + Q_{0,10}(t)[s]\ [1+Y_{10}(t)]$

$Y_1(t) = Q_{10}(t)[s]Y_0(t) + Q_{12}^{(5)}(t)[s]$

$Y_2(t) + Q_{11}^{(4)}(t)]\ [s]Y_1(t)$,

$Y_2(t) = Q_{23}(t)[s]Y_3(t) + Q_{28}^{(7)}(t)\ [s]$

$Y_8(t) + Q_{29}^{(6)}(t)]\ [c]Y_9(t)$

$Y_3(t) = Q_{30}(t)[s][1+Y_0(t)\ ]$

$Y_8(t) = Q_{82}(t)[s]Y_2(t)$

$Y_9(t) = Q_{91}(t)[s]Y_1(t)$

$Y_{10}(t)=Q_{10,0}(t)[s]Y_0(t)+ Q_{10,1}^{(11)}(t)[s]\ Y_1(t) + Q_{10,2}^{(12)}(t)[s]\ Y_2(t)$ (47-53)

Taking Laplace Transform of eq. (47-53) and solving for$Y_0^*(s)$,we get

$Y_0^*(s) = N_6(s) / D_3(s)$ (54)

$N_6(s) = Q_{0,10}^*\ [\{1 - Q_{11}^{(4)*}\}(1- Q_{28}^{(5)*}\ Q_{82}^*\ \} - Q_{12}^{(5)*}Q_{29}^{(6)*}\ Q_{91}^*\{1- Q_{0,10}^*Q_{,10,0}^*\ \}+\{Q_{02}^* + Q_{0,10}^*Q_{10,2}^{(11)*}\}[\ [\ Q_{23}^*\ Q_{30}^*\ \{1 - Q_{11}^{(4)*}\}+ Q_{10}^*\ Q_{29}^{(6)*}\ Q_{91}^*\ ]$

(Omitting the arguments s for brevity)

In the long run,

$W_0 = N_6(0) / D_3'(0)$ (55)

where $N_6(0) = p_{0,10}[\{1-p_{11}^{(4)}\ \}\{1- p_{28}^{(7)}\}- p_{12}^{(5)}\ p_{29}^{(6)}]$

$p_{12}^{(5)} + \{\ p_{02}+ p_{0,10}\ p_{10,2}^{(11}\}\ \{1 - p_{11}^{(4)}\}]$

Benefit- Function Analysis

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure due to bad weather conditions and air traffic congestion caused by miscommunication and failure due to struck by lightning, expected number of visits by the repairman for unit failure. The expected total Benefit-Function incurred in (0,t] is

C(t) = Expected total revenue in (0,t]

- expected busy period of the server when there is failure due to bad weather conditions and air traffic congestion caused by miscommunication and failure due to fog and wind deadliest air disasters caused by miscommunication in (0,t]

- expected number of visits by the repairman Type- I or Type- II for repairing of identical the units in (0,t]

- expected number of visits by the multispecialty repairman Type- III for repairing of identical the units in (0,t]

- expected number of visits by the multispecialty repairman Type- IV for repairing of identical the units in (0,t]

$$C = \lim_{t \to \infty}(C(t)/t) = \lim_{s \to 0}(s^2 C(s))$$

$$= K_1A_0 - K_2R_0 - K_3H_0 - K_4W_0 - K_5Y_0$$

where

K_1 - revenue per unit up-time,

K_2 - cost per unit time for which the system is busy under repairing,

K_3 - cost per visit by the repairman type- I or type- II for units repair,

K_4 - cost per visit by the multispecialty repairman Type- III for units repair

K_5 - cost per visit by the multispecialty repairman Type- IV for units repair

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to failure due to bad weather conditions and air traffic congestion caused by miscommunication, failure due to fog and wind deadliest air disasters caused by miscommunication increases, the MTSF, steady state availability decreases and the Profit-function decreased as the failure increases.

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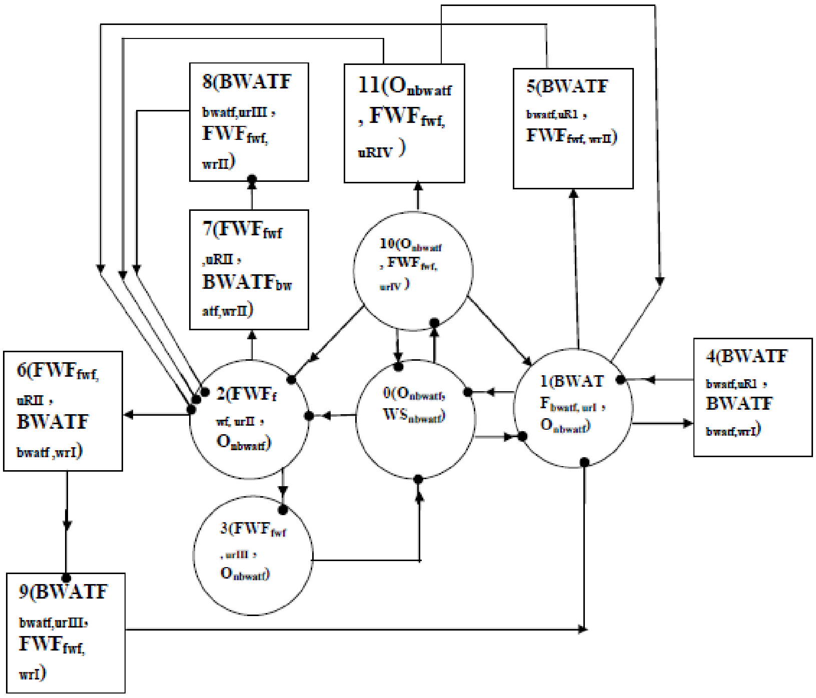

Fig. The State Transition Diagram

○ Up-State □ Down-State

● regeneration point

◆ ◆ ◆