# ANALYSIS OF A HYPOID GEAR
# [Static And Dynamic Analysis]

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**Abstract— The aim of the present article is to delve into some of the concepts related to the Hypoid gears and to check the given gear dimensions. The gear dimensions are checked by doing the static and dynamic analysis. These tests are a basic requirement, as the Hypoid gears assembling test rigs are complex, expensive and is a time consuming process. The test results from the Static and Dynamic analysis (includes Modal and Harmonic Analysis) are studied for conformance with respect to material's Yield stress, Critical frequencies, Effective mass, Mass Participation factors, Deflection and the Von Mises stresses at these Critical frequencies.**

**Keywords— Hypoid gear, Static analysis, Dynamic Analysis, Hypoid offset, Hyperboloid, Axoid, Pitch surface.**

## I. INTRODUCTION

### Gears

Gears are toothed members which transmit power / motion between two shafts by meshing without any slip. When pinion (small gear) is the driver, it results in step down drive in which the output speed decreases and the torque increases. On the other hand, when the gear (big gear) is the driver, it results in step up drive in which the output speed increases and the torque decreases.

When two gears mesh a mechanical advantage is produced, with the rotational speeds and the torques of the two gears differing in an inverse relationship.

### Hypoid gear

Hypoid gears are widely used to transmit crossed-axis power and motion in vehicles, ships and aircrafts. They offer higher load capability and axis position flexibility than spiral bevel gears. A hypoid gear can be considered a cross between a bevel gear and a worm drive. They offer a quiet and smooth meshing of gears.

A hypoid gear is a style of spiral bevel gear whose main variance is that the mating gears' axes do not intersect. The hypoid gear is offset from the gear center, allowing unique configurations and a large diameter shaft. The teeth on a hypoid gear are helical, and the pitch surface is described as a hyperboloid.

Most automotive applications utilize a hypoid gear system, meaning that the gears have an offset. Typical rear axles have a left-hand spiral angle on the pinion gear and a right-hand spiral angle on the ring gear to accommodate a below-centre offset. The hypoid gear system has an offset or difference between the pinion centerline and ring gear centerline. This offset is given as the variable E.

A below-centre offset allows the propeller shaft to be located lower in the vehicle relative to the axle shafts. This allows the tunnel in the vehicle to be shallower and protrude less into the passenger compartment. These pinion-and-ring-gear-spiral-hand and below-centre-offset arrangements are such that the pinion is thrust forward in the vehicle, or toward the head bearing, during forward-drive conditions. This is the main reason that the head bearing is typically larger than the tail bearing.

The pitch surfaces appear conical but, to compensate for the offset shaft, are in fact hyperboloids of revolution. Hypoid gears are almost always designed to operate with shafts at 90 degrees. Depending on which side the shaft is offset to, relative to the angling of the teeth, contact between hypoid gear teeth may be even smoother and more gradual than with spiral bevel gear teeth, but also have a sliding action along the meshing teeth as it rotates. Also, the pinion can be designed with fewer teeth than a spiral bevel pinion, with the result that gear ratios of 60:1 and higher are feasible using a single set of hypoid gears. This style of gear is most commonly found driving mechanical differentials; which are normally straight cut bevel gears; in motor vehicle axles. Their efficiency is lower than other two types of bevel gears. Hence they require the most viscous types of gear oil (HP with number – Hypoid, number for viscosity).

Fig . 1 Hypoid Gear

**Application requirements –**

- **Power, velocity and torque** - consistency and output peaks of the gear drive, so the gear meets mechanical requirements.
- **Inertia -** through acceleration and deceleration. Heavier gears can be harder to stop or reverse.
- **Precision -** requirement of gear, including gear pitch, shaft diameter, pressure angle and tooth layout.
- **Handedness** (left or right teeth angles) – depends on the drive angle. Hypoid gears are usually produced in left-right pairs.
- **Lubrication** - for smooth, temperate operation.
- **Mounting** - may limit the gear's shaft positioning.
- **Noise limitation** – for commercial applications.

## II. DIMENSION SPECIFICATIONS OF HYPOID GEARS

Gears mate via teeth with very specific geometry. Pressure angle is the angle of tooth drive action, or the angle between the line of force between meshing teeth and the tangent to the pitch circle at the point of mesh. Typical pressure angles are 14.5° or 20°, but Hypoid, sometimes operate at 25°. Helix angle is the angle at which the gear teeth are aligned compared to the axis. Hypoid gear arrangements are typically of opposite hands and have larger helical angle.

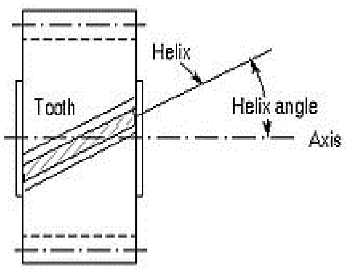



Fig 2 – Gear Tooth Specifications

**Mounting Specifications**

The offset nature of hypoid gears may limit the distance from which the hypoid gear's axis may deviate from the corresponding gear's axis. Offset drives should be limited to 25% of the of the mating gear's diameter, and on heavily loaded alignments should not exceed 12.5% of the mating gear's diameter.

**Accessories**

When hypoid gears are used in vehicle gearboxes, high pressure gear oil is to be used to reduce the friction, wear and heat produced during the sliding action and heavy workloads being taken up by them. Care should be taken if the gearing contains copper, as some high-pressure lubricant additives erode copper.

**Table 1 Design parameters of the Hypoid gear**

| **Parameter** | **Value** |
|---|---|
| Number of Teeth | 43 |
| Proportional top land thickness (mm) | 9 |
| Center distance (mm) | 270 |
| Proportional base tooth thickness (mm) | 90 |
| Profile angle in the hypoid intersection point (deg) | 42.14 |
| Profile angles on outside diameters, deg | 41.23 |
| Operating pressure angle (deg) | 24.98 |
| Transverse contact ratio | 1.6 |
| Profile angles in bottom contact points (deg) | 90 |
| Base diameters (mm) | 43.5 |
| Base pitch (mm) | 18 |
| Operating pitch (mm) | 20 |
| Operating pitch diameters (mm) | 51 |
| Operating tooth thickness (mm) | 18.75 |
| Outside diameters (mm) | 270 |

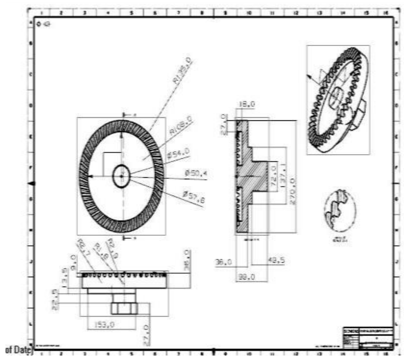


Fig.3 2D drawing of Hypoid gear

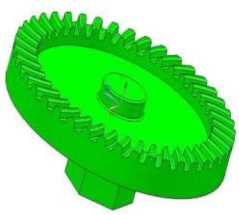


Fig.4 3D model of Hypoid gear (isometric view)

## III. FINITE ELEMENT MODEL

A detailed Finite Element model was developed with solid element 92 to idealize all the components of the Hypoid gear.

**Solid92:**

SOLID92 has quadratic displacement behavior and is well suited to model irregular meshes.

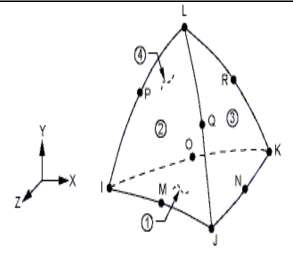
| | |
|---|---|
|  | Element type: Solid92<br>No. of nodes: 10<br>Degrees of freedom: 3<br>(UX, UY, UZ) |

Figure 5: Solid92 geometry and element details

**Material properties: -**

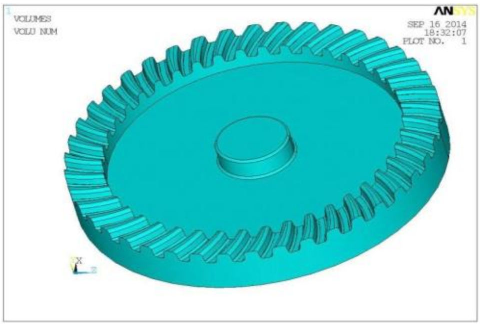
**Hot rolled Steel IS: 2062-1999 (Grade A, Fe 410WA):**

Young's modulus = 200Gpa

Tensile Strength = 410 Mpa

Yield Strength = 250 Mpa

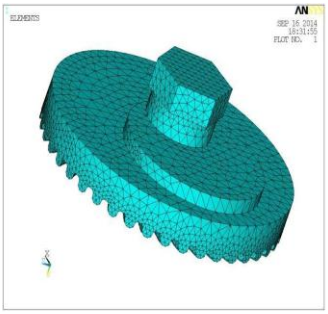
Density = 7850 kg/m3



No. of elements created 50440

No. of nodes created 78978

Fig.6 Geometric model of the Hypoid gear



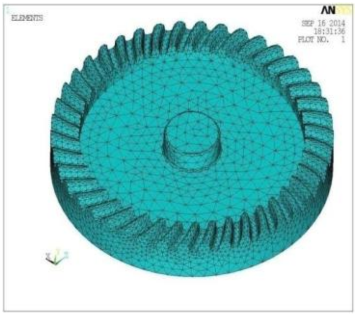
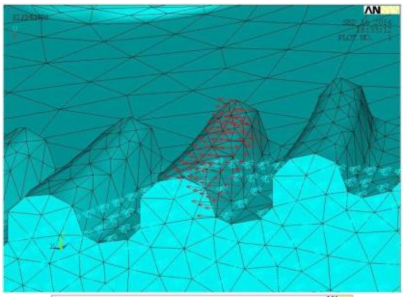


Fig.7 Finite element model of the Hypoid gear

### a) STATIC ANALYSIS OF HYPOID GEAR

Axial force on one teeth at once = 4187N. Hypoid gear is arrested in all Dof at Hexagonal bolt region. From the analysis results the factor of safety of the Hypoid gear at different loca tions are calculated.



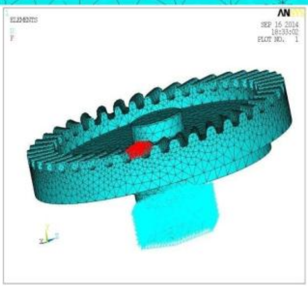


Fig.8 Boundary conditions for static analysis of Hypoid gear

**Results: Deflections – Von Mises Stresses**

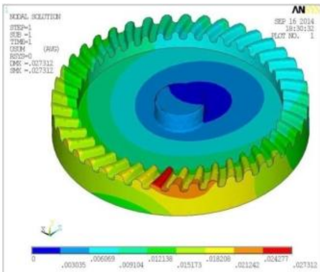


Fig.9 Total Deflection for static analysis

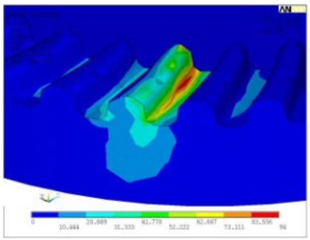


Fig.10 Von Mises stress for static analysis

**Table 2 : Static Analysis results :**

| | |
|---|---|
| Maximum displacement - 0.027mm | Factor of Safety = YS / 94 = 2.6 |
| Max Von Mises stress at 1) Tooth - 104 Mpa 2) Other locations – 94 MPa. | |

### b) DYNAMIC ANALYSIS OF HYPOID GEAR

### i) MODAL ANALYSIS:

In the frequency range of 0 -2000 Hz.

The objective of the analysis is to perform the modal analysis of the Hypoid gear and find the natural frequencies. It serves as a starting point for Harmonic Analysis.

**Natural Frequency:**

Natural frequency is the frequency at which a system naturally vibrates once it has been set into motion. The natural frequencies depend on stiffness of the geometry and mass of the material and is calculated from the formula given below.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

**Fundamental Natural Frequency**

The fundamental frequency is the lowest frequency of a periodic waveform. In terms of a superposition of sinusoids (e.g. Fourier series), the fundamental frequency is the lowest frequency sinusoidal in the sum.

**Resonance:**

The system oscillates at higher amplitudes due to resonance at some frequencies than at others, because the system stores vibrational energy.

Resonance occurs when a system is able to store and easily transfer energy between two or more different storage modes. However, there are some losses from cycle to cycle, called damping. When damping is small, the resonant frequency is approximately equal to the natural frequency of the system, which is a frequency of unforced vibrations.

**Mode Shapes:**

For every natural frequency there is a corresponding vibration mode shape. Most mode shapes can generally be described as being an axial mode, torsional mode, bending mode, or general modes. Like stress analysis models, probably the most challenging part of getting accurate finite element natural frequencies and mode shapes is to get the type and locations of the restraints correct. A crude mesh will give accurate frequency values, but not accurate stress values.

The total weight of the crank case is 142kg.The mode shapes and the mass participation of each of these 6 frequencies are given below,

**Boundary Conditions:**

Hypoid gear is arrested in all Dof and at Hexagonal bolt region.

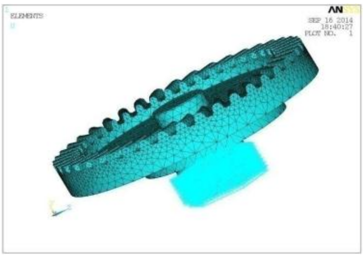
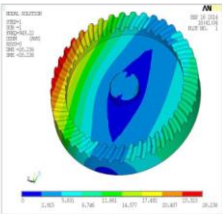


Fig.11 Applied Boundary conditions on Hypoid gear

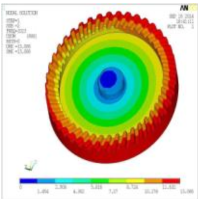
**Table 3 : Frequencies and mass participation**

| MODE | FREQUENCY | PARTIC. FACTOR (EFFECTIVE MASS) | | |
|---|---|---|---|---|
| | | X- Dir | Y- Dir | Z- Dir |
| 1 | 949 | -0.2E-01 (0.7 E-03) | 0.2E-01 (0.5E-03) | 0.1E-02 (0.2E-05) |
| 2 | 1022 | 0.5E-03 (0.2E-06) | -0.5E-05 (0.3E-10) | 0.1E-01 (0.1E-03) |
| 3 | 1051 | 0.16E-02 (0.3E-05) | 0.15E-04 (0.2E-09) | 0.3E-01 (0.1E-02) |
| 4 | 1521 | 0.28E-01 (0.8E-03) | 0.8E-01 (0.7E-02) | -0.1E-02 (0.1E-05) |
| 5 | 1740 | 0.88E-03 (0.7E-06) | 0.7E-04 (0.5E-08) | 0.2E-01 (0.3E-03) |
| 6 | 1782 | -0.3E-02 (0.1E-04) | 0.3E-01 (0.1E-02) | -0.3E-04 (0.1E-08) |

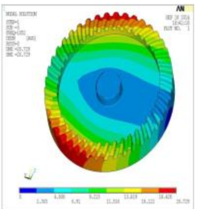
**Mode Shapes :**



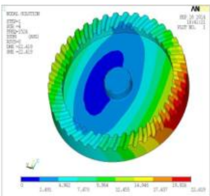
@ 949Hz



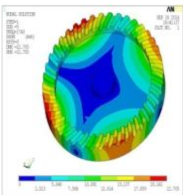
@ 1023 Hz



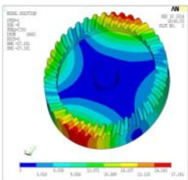
@ 1052Hz



@ 1524Hz



@ 1740Hz



@ 1783Hz

**Table 4 : Frequencies and Mass Participation for Modes (range of 0-2000Hz).**

**The max. mass participation is indicated in color at the respective frequency.**

| MODE | FREQUENCY | PARTIC. FACTOR (EFFECTIVE MASS) | | |
|---|---|---|---|---|
| | | X- Dir | Y- Dir | Z- Dir |
| 1 | 949 | -0.2E-01 (0.7 E-03) | 0.2E-01 (0.5E-03) | 0.1E-02 (0.2E-05) |
| 3 | 1051 | 0.16E-02 (0.3E-05) | 0.15E-04 (0.2E-09) | 0.3E-01 (0.1E-02) |
| 4 | 1521 | 0.28E-01 (0.8E-03) | 0.8E-01 (0.7E-02) | -0.1E-02 (0.1E-05) |

**ii) HARMONIC ANALYSIS OF HYPOID GEAR**

To overcome the effects of resonance, fatigue and other effects of forced vibrations during the sustained cyclic loading in the structural system, Harmonic response analysis is to be done.

Harmonic response analysis is a technique used to determine the steady-state response of a linear structure to loads that vary sinusoidally (harmonically) with time. The idea is to calculate the structure's response at several frequencies and obtain a graph of some response quantity (usually displacements) versus frequency. "Peak" responses are then identified on the graph and stresses reviewed at those peak frequencies.

The procedure for a full harmonic response analysis consists of three main steps:

1. Build the model.
2. Apply loads and obtain the solution.
3. Review the results.

Harmonic response occurs at forcing frequencies that match the natural frequencies of the structure.

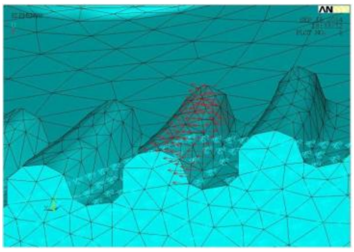
To completely specify a harmonic load, the information about the amplitude, the phase angle, and the forcing frequency range are required.

Deflections and stress of a structure in the frequency range of 0 -2000 Hz are recorded and plotted.

No. Of sub steps = 10.

**Boundary conditions for harmonic analysis:**

Axial force on one teeth at once = 4187N. Hypoid gear is arrested in all Dof and at Hexagonal bolt region.



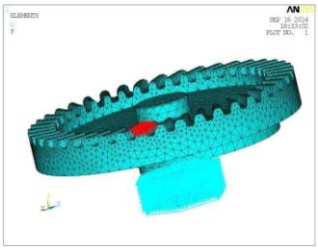


Fig.12 Boundary conditions applied for harmonic analysis of Hypoid gear

**Table 5 : MODAL ANALYSIS RESULTS:**

**Freq. & Mass Participation factors**

| MODE | FREQUENCY | PARTIC. FACTOR (MASS PARTICIPATION FACTOR) | | |
|---|---|---|---|---|
| | | X- Dir | Y- Dir | Z- Dir |
| 1 | 949 | -0.2E-01 (0.7 E-03) | 0.2E-01 (0.5E-03) | 0.1E-02 (0.2E-05) |
| 3 | 1051 | 0.16E-02 (0.3E-05) | 0.15E-04 (0.2E-09) | 0.3E-01 (0.1E-02) |
| 4 | 1521 | 0.28E-01 (0.8E-03) | 0.8E-01 (0.7E-02) | -0.1E-02 (0.1E-05) |

**HARMONIC ANALYSIS RESULTS**

**GRAPHS: AMPLITUDE $V_S$ FORCING FREQUENCY:**

1. **Harmonic response at fixed end location**

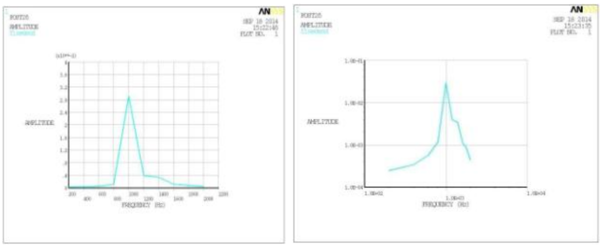


Fig.13 Linear scale log scale

**Harmonic response at gear centre –**

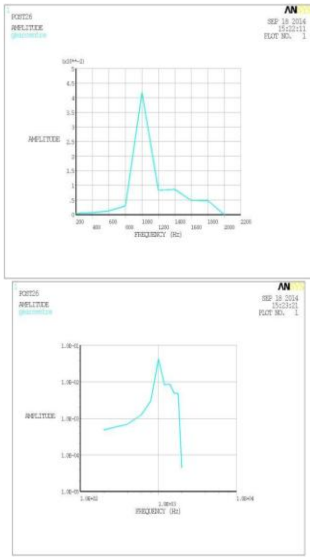
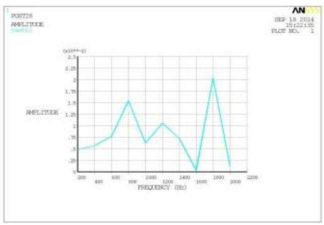


Fig.14 Linear scale log scale

**Harmonic response at teeth –**



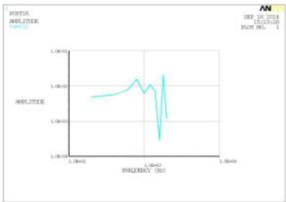


Fig.15 Linear scale log scale

**Table 6 : From the above graphs, the following amplitudes are observed –**

| Amplitude (mm) | Location | Frequency (Hz) |
|---|---|---|
| 2.9 | At the Fixed end | 1000 |
| 4.2 | At Gear center | 1000 |
| 2.0 | On the Gear teeth | 1800 |

**DEFLECTIONS AND STRESSES**

1. **Max. Deflection and stress of frequency @ 1000Hz**

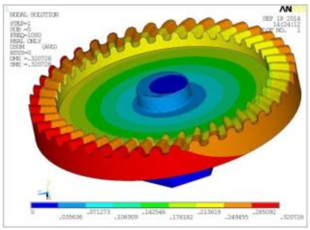


Fig.16 Max. Deflection

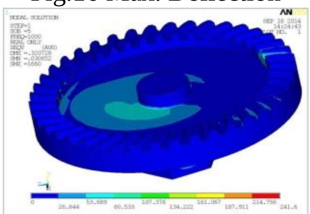


Fig.17 Von Mises stress

2. **Max. Deflection and stress of frequency @ 1800Hz**

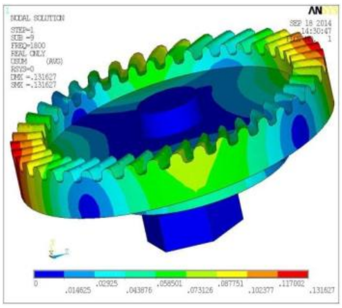


Fig.18 Max. Deflection

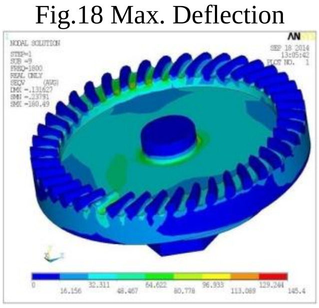


Fig.19 Von Mises stress

**Table 7: The deflections and Von Mises stress for critical frequencies -**

| S. No | FREQUENCY (Hz) | DEFLECTIONS (mm) | VON MISES STRESS (MPa) |
|---|---|---|---|
| 1 | 1000 | 0.32 | 241.4 |
| 2 | 1800 | 0.13 | 145.4 |

The design of given Hypoid gear is acceptable as the Von Mises stress of Hypoid gear at frequencies are less than the yield strength of the material.

## IV. RESULTS AND DISCUSSIONS

**The Hypoid gear was studied for 3 different cases and following observations are done:**

**a) Static Analysis:**

According to the Maximum Yield Stress Theory, the VonMises stress (104 MPa) is less than the yield strength (250 MPa) of the material. Hence the design of Hypoid gear is safe for the above operating loads.

**b) Dynamic Analysis**

**i) Modal Analysis:**

It is observed that the maximum mass participation of 0.0007tonne is observed in X-dir for the frequency of 949Hz.

It is observed that the maximum mass participation of 0.007tonne is observed in Y-dir for the frequency of 1521Hz.

It is observed that the maximum mass participation of 0.001tonne is observed in Z-dir for the frequency of 1051Hz.

**ii) Harmonic Analysis:**

From the results it is observed that the critical frequencies 1000 Hz and 1800 Hz are having stresses of 241MPa and 145 MPa respectively. The yield strength of the material used for Hypoid gear is 250 MPa.

Hence the design of Hypoid gear is safe for the above operating loading conditions.

## V. CONCLUSION

It is concluded that the Hypoid gear is safe under the given operating conditions.

## VI. FUTURE SCOPE

1. All 6 d.o.f.s of the gear body are to be modelled to explicitly predict accurate results.

2. Dynamic analysis of the gear due to the effect of the rolling elements of the bearings.

3. The analysis can be studied for different materials.

4. Analysis of the gear due to the effect of temperature while working under severe conditions.

5. The effects of cutter radius on tooth proportions.

## ACKNOWLEDGMENT

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