



Performance analysis of various sub-systems of process plant using markov chain method

¹Navkiran Singh, ²Atul Goyal

^{1,2}Deptt. of Mechanical Engineering, LLRIET, Moga

Abstract: Traditional engineering methods for the performance evaluation of manufacturing systems assume that machine reliability parameters (Mean Time to Failure and Mean Time to Repair) are precisely known. The present study entitled “Performance Evaluation of Beverage Industry in India”- A Case Study of Bilaspur Beverage Private Limited Industry in India has been undertaken with the object of analyzing and evaluating the availability of plant. Studies in performance evaluation of automated manufacturing systems, using simulation or analytical models, have always emphasized steady-state or equilibrium performance in preference to transient performance. Manufacturing systems and models in which such situations arise include: systems with failure states and deadlocks, unstable queueing systems. This paper proposes an approach for the performance evaluation of unreliable manufacturing systems that considers uncertain machine parameter estimates. The proposed method is based on the markov chain and probability density function discretization techniques for analyzing manufacturing lines composed of unreliable machines. In order to analysis the performance of plant, most useful information has been gathered from the various systems and sub systems to calculate long run availability of whole system.

Keywords: Performance Evaluation, Steady State Probabilities, Markov Approach, Transient Analysis.

I. INTRODUCTION

Manufacturing flow line systems consist of material, work areas, and storage areas. Material flows from work area to storage area to work area. It visits each work and storage area exactly once in a fixed sequence, there is a first work area through which material enters and a last work area through which it leaves the system. Manufacturing flow lines are also called transfer lines. In this paper, we mainly use the term ‘reliability’, in general can be defined as the probability of a system or device performing its anticipated purpose adequately for the intended period of time under the given operating conditions. Reliability engineering had gained its importance in recent years due to the good results. Today the industries are of high concern with the safety of their machines and also the uninterrupted working of the system. So, the reliability engineering is an important tool to compute and improve the systems performance which is widely used now a days. Recent studies by the researchers in the field of reliability/

availability/ maintainability proposed several methods for industrial systems under maintenance. Reliability and availability analysis can benefit the industry in terms of higher productivity and lower maintenance cost which is possible to improve the availability of the plant with proper maintenance, planning, monitoring and control. In fact, uninterrupted operation is an essential requirement of large complex systems.

II. SYSTEM DESCRIPTION

In this system, the prepared hot beverage is filled in the rinsed bottles, where bottles are warmed by spraying hot water before filling to prevent the cracks and then, filled bottles are sealed in the next unit. The hot fill bottles need to cool down at normal room temperature, so that they are passed through cooling tunnel, which is divided into four sections from higher temperature to lower temperature region. When bottles are entered into the tunnel, the water is sprayed around 55-50°C. Temperature further declines in next stage up to 50-45°C and it reaches the minimum temperature at the last exit stage, it is around 35-30°C. After leaving the unit, manufacturing date is printed on bottles by jet spray machine. Further, bottles are settled down on the crates which is self-supporting rectangular structure, often made of wood, used for shipping of items from one place to other place for dispatching.

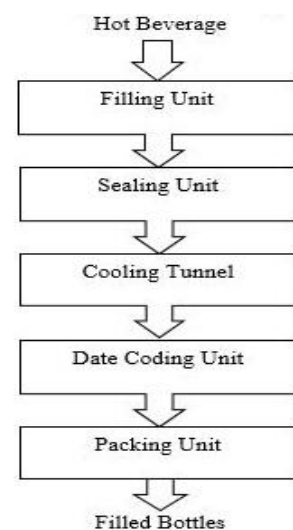


Fig. 1 Process flow Diagram

III. NOTATIONS AND ASSUMPTIONS

Notations

Filling unit (F): One unit subjected to major failure only.

Sealing unit (S): One unit subjected to major failure only.

Cooling unit (C): One unit subjected to major failure only.

Date coding unit (D): One unit subjected to major failure only.

Packing unit (P): One unit subjected to major failure only.

λ_i : Failure rate of F,S,C,D & P units,(i = 1,2,3,4,5).

μ_i = Repair rate of F,S,C,D & P units,(i = 1,2,3,4,5).

0: Represents the system/sub-system is operating.

r: Component/sub-system is under repair.

g: Component is working in good condition.

Assumptions

- All the subsystems are initially operating.
- All the sub-systems are initially in good state.
- Each unit has two states viz., good and failed.
- It is also assumed that there is only one repair facility and priority will be given to the sub-system F, S, C, D and P for repair.
- Each unit is good as new after repair.
- The failure rates and repair rates of all units are taken constant.
- Failure and repair events are statistically independent.

IV. MATHEMATICAL ANALYSIS OF THE SYSTEM

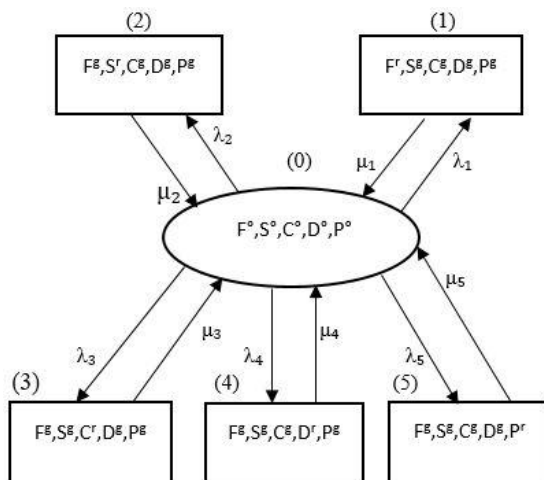
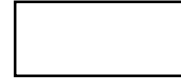


Fig. 2 System state transition diagram



System working
at full capacity



System failed
state

Probability consideration gives the following first order differential-difference equations associated with the state transition diagram of the system.

$$P_0(t) + (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)P_0(t) = \mu_1P_1(t) + \mu_2P_2(t) + \mu_3P_3(t) + \mu_4P_4(t) + \mu_5P_5(t)$$

$$P_0(t) + \alpha_1P_0(t) = \mu_1P_1(t) + \mu_2P_2(t) + \mu_3P_3(t) + \mu_4P_4(t) + \mu_5P_5(t) \quad (1)$$

Where $\alpha_1 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5$

$$P_1(t) + \mu_1P_1(t) = \lambda_1P_0(t) \quad (2)$$

$$P_2(t) + \mu_2P_2(t) = \lambda_2P_0(t) \quad (3)$$

$$P_3(t) + \mu_3P_3(t) = \lambda_3P_0(t) \quad (4)$$

$$P_4(t) + \mu_4P_4(t) = \lambda_4P_0(t) \quad (5)$$

$$P_5(t) + \mu_5P_5(t) = \lambda_5P_0(t) \quad (6)$$

With initial condition at time $t = 0$

$$P_i(t) = 1 \quad \text{for } i = 0$$

$$P_i(t) = 0 \quad \text{for } i \neq 0$$

Solution of equations

Taking Laplace Transformation of above equations (2), (3), (4), (5) and (6), we get

$$P_i(s) = K_iP_0(s) \quad \text{For } i = 1 \text{ to } 5$$

Where

$$K_1 = \frac{\lambda_1}{s + \mu_1}, \quad K_2 = \frac{\lambda_2}{s + \mu_2}$$

$$K_3 = \frac{\lambda_3}{s + \mu_3}, \quad K_4 = \frac{\lambda_4}{s + \mu_4}, \quad K_5 = \frac{\lambda_5}{s + \mu_5}$$

Taking Laplace Transformation of equations (1) using initial condition

$$sP_0(s) + \alpha_1P_0(s) = 1 + \mu_1K_1P_0(s) + \mu_2K_2P_0(s) + \mu_3K_3P_0(s) + \mu_4K_4P_0(s) + \mu_5K_5P_0(s)$$

$$(s + \alpha_1)P_0(s) = 1 + P_0(s)(\mu_1K_1 + \mu_2K_2 + \mu_3K_3 + \mu_4K_4 + \mu_5K_5)$$

$$P_0(s) = \{(s + \alpha_1) - (\mu_1K_1 + \mu_2K_2 + \mu_3K_3 + \mu_4K_4 + \mu_5K_5)\}^{-1} \quad (7)$$

Laplace transformation of availability function $A(t)$ is give as

$$A(s) = P_0(s)$$

Inversion of $A(s)$ gives the availability function $A(t)$

Now applying steady state condition on first order differential-difference equation

$$\text{When } t \rightarrow \infty, \quad \frac{d}{dt} \rightarrow 0,$$

From equation (1),

$$\alpha_1 P_0 = \mu_1 P_1 + \mu_2 P_2 + \mu_3 P_3 + \mu_4 P_4 + \mu_5 P_5$$

Similarly, from equation (2) to (6)

$$\mu_1 P_1 = \lambda_1 P_0$$

$$\mu_2 P_2 = \lambda_2 P_0$$

$$\mu_3 P_3 = \lambda_3 P_0$$

$$\mu_4 P_4 = \lambda_4 P_0$$

$$\mu_5 P_5 = \lambda_5 P_0$$

Further reducing these equations, we get

$$P_i = L_i P_0 \quad \text{Where } i = 1 \text{ to } 5$$

Where

$$L_i = \lambda_i / \mu_i$$

Using normalizing equation, when sum of all the probability is equal to one i.e.

$$\sum_{i=0}^5 P_i = 1$$

$$P_0 + P_1 + P_2 + P_3 + P_4 + P_5 = 1$$

$$P_0 = \left[1 + \sum_{i=1}^5 \frac{\lambda_i}{\mu_i} \right]^{-1} \quad (8)$$

The Overall steady state availability of the system when running at full capacity is

$$AFC = P_0$$

Where P_0 is given by equation (8)

$$AFC = \left[1 + \sum_{i=1}^5 \frac{\lambda_i}{\mu_i} \right]^{-1} \quad (9)$$

V. PERFORMANCE ANALYSIS

The effects of failure rate, repair rate and maintenance rate of various components and sub components comprising the system are examined and their impact are described in the below tables.

Results are obtained for performance analysis of frooti preparation system.

(A) Effect of failure rate of filling unit on availability AFC: Taking $\lambda_2 = 0.01$, $\lambda_3 = 0.012$, $\lambda_4 = 0.006$, $\lambda_5 = 0.005$, $\mu_1 = 0.25$, $\mu_2 = 0.20$, $\mu_3 = 0.25$, $\mu_4 = 0.35$, $\mu_5 = 0.125$

Table 1 Steady state availability versus failure rate of filling unit.

λ_1	0.004	0.006	0.008	0.010
AFC	0.8539	0.8482	0.8424	0.8368

(B) Effect of failure rate of sealing unit on availability AFC: Taking $\lambda_1 = 0.004$, $\lambda_3 = 0.012$, $\lambda_4 = 0.006$, $\lambda_5 = 0.005$, $\mu_1 = 0.25$, $\mu_2 = 0.20$, $\mu_3 = 0.25$, $\mu_4 = 0.35$, $\mu_5 = 0.125$

Table 2 Steady state availability versus failure rate of sealing unit.

λ_2	0.01	0.02	0.03	0.04
AFC	0.8539	0.8190	0.7867	0.7570

(C) Effect of failure rate of cooling tunnel on availability AFC: Taking $\lambda_1 = 0.004$, $\lambda_2 = 0.01$, $\lambda_4 = 0.006$, $\lambda_5 = 0.005$, $\mu_1 = 0.25$, $\mu_2 = 0.20$, $\mu_3 = 0.25$, $\mu_4 = 0.35$, $\mu_5 = 0.125$

Table 3 Steady state availability versus failure rate of cooling tunnel.

λ_3	0.012	0.014	0.016	0.018
AFC	0.8539	0.8481	0.8424	0.8368

(D) Effect of failure rate of date coding unit on availability AFC: Taking $\lambda_1 = 0.004$, $\lambda_2 = 0.01$, $\lambda_3 = 0.012$, $\lambda_5 = 0.005$, $\mu_1 = 0.25$, $\mu_2 = 0.20$, $\mu_3 = 0.25$, $\mu_4 = 0.35$, $\mu_5 = 0.125$

Table 4 Steady state availability versus failure rate of date coding unit.

λ_4	0.006	0.008	0.010	0.012
AFC	0.8539	0.8497	0.8456	0.8415

(E) Effect of failure rate of packing unit on availability AFC: Taking $\lambda_1 = 0.004$, $\lambda_2 = 0.01$, $\lambda_3 = 0.012$, $\lambda_4 = 0.006$, $\mu_1 = 0.25$, $\mu_2 = 0.20$, $\mu_3 = 0.25$, $\mu_4 = 0.35$, $\mu_5 = 0.125$

Table 5 Steady state availability versus failure rate of packing unit.

λ_5	0.005	0.007	0.009	0.011
AFC	0.8539	0.8424	0.8313	0.8203

(F) Effect of repair rate of filling unit on availability AFC: Taking $\lambda_1 = 0.004$, $\lambda_2 = 0.01$, $\lambda_3 = 0.012$, $\lambda_4 = 0.006$, $\lambda_5 = 0.005$, $\mu_2 = 0.20$, $\mu_3 = 0.25$, $\mu_4 = 0.35$, $\mu_5 = 0.125$

Table 6 Steady state availability versus repair rate of filling unit.

μ_1	0.25	0.50	0.75	1.0
AFC	0.8539	0.8598	0.8618	0.8628

(G) Effect of repair rate of sealing unit on availability AFC: Taking $\lambda_1 = 0.004$, $\lambda_2 = 0.01$, $\lambda_3 = 0.012$, $\lambda_4 = 0.006$, $\lambda_5 = 0.005$, $\mu_1 = 0.25$, $\mu_3 = 0.25$, $\mu_4 = 0.35$, $\mu_5 = 0.125$

Table 7 Steady state availability versus repair rate of sealing unit.

μ_2	0.20	0.30	0.40	0.50
AFC	0.8539	0.8663	0.8726	0.8764

(H) Effect of repair rate of cooling tunnel on availability AFC: Taking $\lambda_1 = 0.004$, $\lambda_2 = 0.01$, $\lambda_3 = 0.012$, $\lambda_4 = 0.006$, $\lambda_5 = 0.005$, $\mu_1 = 0.25$, $\mu_2 = 0.20$, $\mu_4 = 0.35$, $\mu_5 = 0.125$

Table 8 Steady state availability versus repair rate of cooling tunnel.

μ_3	0.25	0.50	0.75	1.0
AFC	0.8539	0.8718	0.8779	0.8810

(I) Effect of repair rate of date coding unit on availability AFC: Taking $\lambda_1 = 0.004$, $\lambda_2 = 0.01$, $\lambda_3 = 0.012$, $\lambda_4 = 0.006$, $\lambda_5 = 0.005$, $\mu_1 = 0.25$, $\mu_2 = 0.20$, $\mu_3 = 0.25$, $\mu_5 = 0.125$

Table 9 Steady state availability versus repair rate of date coding unit.

μ_4	0.35	0.50	0.65	0.80
AFC	0.8539	0.8576	0.8596	0.8609

(J) Effect of repair rate of packing unit on availability AFC: Taking $\lambda_1 = 0.004$, $\lambda_2 = 0.01$, $\lambda_3 = 0.012$, $\lambda_4 = 0.006$, $\lambda_5 = 0.005$, $\mu_1 = 0.25$, $\mu_2 = 0.20$, $\mu_3 = 0.25$, $\mu_4 = 0.35$

Table 10 Steady state availability versus repair rate of packing unit.

μ_5	0.125	0.250	0.375	0.5
AFC	0.8539	0.8688	0.8738	0.8764

VI. RESULTS AND DISCUSSION

The effect of failure and repair rates of various sub-systems on the availability for transient state is examined. From the results obtained through performance analysis, it is found that increase in the failure rate of filling unit, sealing unit, cooling unit, date

coding unit and packing unit reduce the availability of the plant to greater extent. On the other side, with increase in repair rate, the availability of the plant is increased by 0.89%, 2.25%, 2.71%, 0.7%, 2.25% of filling unit, sealing unit, cooling unit, date coding unit and packing unit on increasing repair rate from 0.25 to 1.0, 0.20 to 0.50, 0.25 to 1.0, 0.35 to 0.80, 0.125 to 0.5 respectively.

REFERENCES

- [1] Tolio, T., Matta, A., 1997. A Method for Performance Evaluation of Automated Flow Lines, CIRP Annals, 47/1, p. 373.
- [2] Denardo, E.V., Tang, C.S., 1997. Control of a Stochastic Production System with Estimated Parameters, Management Science, 13(9), p. 1296.
- [3] Lin, L., Ni, J., 2008. Reliability estimation based on operational data of manufacturing systems, Quality and Reliability Engineering International, 24(7), p. 843.
- [4] Parida, N., 1991. Reliability and life estimation from component fatigue failures below the go-no-go fatigue limit, Journal of Testing and Evaluation, 19, p. 450.
- [5] Inman, R.R., 1999. Empirical evaluation of exponential and independence assumptions in queuing models of manufacturing systems, Production and Operations Management, Vol 8(4), p. 409.
- [6] N.Viswanadham and Y. Narahari. Performance Modeling of Automated Manufacturing Systems. Prentice Hall, Englewood Cliffs, NJ, 1992.
- [7] Y. Dallery and S.B. Gershwin. Manufacturing flow line systems: A review of models and analytical results. Technical Report 91-002, Laboratory for Manufacturing and Productivity, MIT, April 1992.
- [8] J.A. Buzacott and D.D. Yao. Flexible manufacturing systems: A review of analytical models. Management Science, 32:890-905, 1986.
- [9] A.Tayal, A. Bhardawaj, A. Goyal „System Modelling & Availability Analysis of Cement Blending Unit Case Study” ISSN (Print): 2321-5747, Volume-2, Issue-2, 2014.
- [10] P. Gupta, J. Singh and I. P. Singh, Maintenance Planning based on Performance Analysis of 7-out-of-14: G Chemical-system: a Case Study, International Journal of Industrial Engineering, 2005, pp. 264-274.
- [11] Kołowrocki, K., "On limit reliability functions of large multi-state systems with ageing components", Appl Math Comput., 2001, Vol.121, pp. 313-61.

- [12] Singh I.P., "A Complex System having Four Types of Components with Preventive Repeat Priority Repairs", *Micron reliability*, 1989, Vol. 29, No. 6, pp. 959-962.
- [13] Misra, K.B. , "Reliability analysis and prediction", Amsterdam: Elsevier.,1992
- [14] R.T. Islamov, "Using Markov Reliability Modelling for Multiple Repairable Systems", *Reliability engineering and system safety*, 1994, pp. 113 – 118.
- [15] Sachin, K. and Anand, T. , "Evaluation of some reliability parameters of a three state repairable system with environmental failure", *International Journal of Research and Reviews in Applied Sciences*, 2009, Vol. 2(1),pp. 96-103.

