Benefit-Function of Two- Identical Cold Standby Nuclear Reactors System subject to failure due to radioactivity or Overheating, steam explosion, fire, and meltdown

Ashok Kumar Saini

BLJS COLLEGE, TOSHAM (BHIWANI) HARYANA INDIA
Email ID drashokksaini2009@gmail.com

Abstract- A nuclear and radiation accident is defined by the International Atomic Energy Agency as "an event that has led to significant consequences to people, the environment or the facility." Examples include lethal effects to individuals, large radioactivity release to the environment, or reactor core melt." The prime example of a "major nuclear accident" is one in which a reactor core is damaged and significant amounts of radioactivity are released, such as in the Chernobyl disaster in 1986.

The impact of nuclear accidents has been a topic of debate practically since the first nuclear reactors were constructed in 1954. It has also been a key factor in public concern about nuclear facilities. Some technical measures to reduce the risk of accidents or to minimize the amount of radioactivity released to the environment have been adopted. Despite the use of such measures, human error remains, and "there have been many accidents with varying impacts as well near misses and incidents".

Worldwide there have been 99 accidents at nuclear power plants. Fifty-seven accidents have occurred since the Chernobyl disaster, and 57% (56 out of 99) of all nuclear-related accidents have occurred in the USA. Serious nuclear power plant accidents include the Fukushima Daiichi nuclear disaster (2011), Chernobyl disaster (1986), Three Mile Island accident (1979), and the SL-1 accident (1961). Nuclear advocate Stuart Arm maintains that, "apart from Chernobyl, no nuclear workers or members of the public have ever died as a result of exposure to radiation due to a commercial nuclear reactor incident."

Nuclear-powered submarine core meltdown and other mishaps include the K-19 (1961), K-11 (1965), K-27 (1968), K-140 (1968), K-429 (1970), K-222 (1980), K-314 (1985), and K-431 (1985). Serious radiation accidents include the Kyshtym disaster, Windscale fire, radiotherapy accident in Costa Rica, radiotherapy accident in Zaragoza, radiation accident in Morocco, Goiania accident, radiation accident in Mexico City, radiotherapy unit accident in Thailand, and the Mayapuri radiological accident in India.

The International Atomic Energy Agency maintains a website reporting recent accidents.

One of the worst nuclear accidents to date was the Chernobyl disaster which occurred in 1986 in Ukraine. The accident killed 30 people directly and damaged approximately $7 billion of property. A study published in 2005 estimates that there will eventually be up to 4,000 additional cancer deaths related to the accident among those exposed to significant radiation levels. Radioactive fallout from the accident was concentrated in areas of Belarus, Ukraine and Russia. Approximately 350,000 people were forcibly resettled away from these areas soon after the accident.

Benjamin K. Sovacool has reported that worldwide there have been 99 accidents at nuclear power plants from 1952 to 2009 (defined as incidents that either resulted in the loss of human life or more than US$50,000 of property damage, the amount the US federal government uses to define major energy accidents that must be reported), totaling US$20.5 billion in property damages. Fifty-seven accidents have occurred since the Chernobyl disaster, and almost two-thirds (56 out of 99) of all nuclear-related accidents have occurred in the US. There have been comparatively few fatalities associated with nuclear power plant accidents.

In this paper we have taken failure due to radioactivity or Overheating, steam explosion, fire, and meltdown. When the main unit fails due to Overheating, steam explosion, fire, and meltdown then cold standby system becomes operative. Overheating, steam explosion, fire, and meltdown cannot occur simultaneously in both the units and after failure the unit undergoes very costly repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

Keywords: Cold Standby, Overheating, steam explosion, fire, and meltdown, first come first serve, MTSF, Availability, Busy period, Benefit -Function.

INTRODUCTION

Serious radiation and other accidents and incidents include: 1940s

- May 1945: Albert Stevens was the subject of a human radiation experiment, and was injected with plutonium without his knowledge or informed consent. Although Stevens was the person who received the highest dose of radiation during the plutonium experiments, he was neither the first nor the last subject to be studied. Eighteen people aged 4 to 69 were injected with plutonium. Subjects who were chosen for the experiment had been diagnosed with a terminal disease. They lived from 6 days up to 44 years past the time of their injection. Eight of the 18 died within 2 years of the injection. All died from their preexisting terminal illness, or cardiac illnesses. None died from the plutonium itself. Patients from Rochester, Chicago, and Oak Ridge were also injected with plutonium in the Manhattan Project human experiments.

- 6–9 August 1945: On the orders of President Harry S. Truman, a uranium-gun design bomb, Little Boy, was used against the city of Hiroshima, Japan. Fat Man, a plutonium implosion-design bomb was used against the city of Nagasaki. The two weapons killed approximately 120,000 to 140,000 civilians and military personnel instantly and thousands more have died over the years from radiation sickness and related cancers.
- August 1945: Criticality accident at US Los Alamos National Laboratory. Harry K. Daghlian, Jr., dies. May 1946: Criticality accident at Los Alamos National Laboratory. Louis Slotindies. 1950s
- February 13, 1950 : a Convair B-36B crashed in northern British Columbia after jettisoning a Mark IV atomic bomb. This was the first such nuclear weapon loss in history.
- December 12, 1952: NRX AECL Chalk River Laboratories, Chalk River, Ontario, Canada. Partial meltdown, about 10,000 Curies released. Approximately 1202 people were involved in the two year cleanup. President Jimmy Carter was one of the many people that helped clean up the accident. 15/03/1953 – Mayak, Former Soviet Union. Criticality accident. Contamination of plant personnel occurred.
- 1954: The 15 Mt Castle Bravo shot of 1954 which spread considerable nuclear fallout on many Pacific islands, including several which were inhabited, and some that had not been evacuated.
- March 1, 1954: Daigo Fukuryū Maru, 1 fatality.
- September 1957: a plutonium fire occurred at the Rocky Flats Plant, which resulted in the contamination of Building 71 and the release of plutonium into the atmosphere, causing US $818,600 in damage.
- 21/04/1957 - Mayak, Former Soviet Union. Criticality accident in the factory number 20 in the collection oxalate decantate after filtering sediment oxalate enriched uranium. Six people received doses of 300 to 1,000 rem (four women and two men), one woman died. September 1957: Kyshtym disaster: Nuclear waste storage tank explosion at Chelyabinsk, Russia. 200+ fatalities, believed to be a conservative estimate; 270,000 people were exposed to dangerous radiation levels. Over thirty small communities were removed from Soviet maps between 1958 and 1991. (INES level 6) October 1957: Windscale fire, UK. Fire ignites plutonium piles and contaminates surrounding dairy farms. An estimated 33 cancer deaths.
- 1957-1964: Rocketdyne located at the Santa Susanna Field Lab, 30 miles north of Los Angeles, California operated ten experimental nuclear reactors. Numerous accidents occurred including a core meltdown. Experimental reactors of that era were not required to have the same type of containment structures that shield modern nuclear reactors. During the Cold War time in which the accidents that occurred at Rockedyne, these events were not publicly reported by the Department of Energy. 10/02/1958 - Mayak, Former Soviet Union. Criticality accident in SCR plant. Conducted experiments to determine the critical mass of enriched uranium in a cylindrical container with different concentrations of uranium in solution. Staff broke the rules and instructions for working with YADM (nuclear fissile material). When SCR personnel received doses from 7600 to 13,000 rem. Three people died, one man got radiation sickness and went blind.
- December 30, 1958: Cecil Kelley criticality accident at Los Alamos National Laboratory.
- March 1959: Santa Susana Field Laboratory, Los Angeles, California. Fire in a fuel processing facility.
- July 1959: Santa Susana Field Laboratory, Los Angeles, California. Partial meltdown. 1960s
- 7 June 1960: the 1960 Fort Dix IM-99 accident destroyed a CIM-10 Bomarc nuclear missile and shelter and contaminated the BOMARC Missile Accident Site in New Jersey.
- 24 January 1961: the 1961 Goldsboro B-52 crash occurred near Goldsboro, North Carolina. A B-52 Stratofortress carrying two Mark 39 nuclear bombs broke up in mid-air, dropping its nuclear payload in the process.
- accident. Eight fatalities and more than 30 people were over-exposed to radiation.
- March, 21 -August 1962: radiation accident in Mexico City, four fatalities.
- May 1962: The Cuban missile crisis was a 13-day confrontation in October 1962 between the Soviet Union and Cuba on one side and the United States on the other side. The crisis is generally regarded as the moment in which the Cold Warcame closest to turning into a nuclear conflict and is also the first documented instance

of mutual assured destruction (MAD) being discussed as a determining factor in a major international arms agreement.

- 1964, 1969: Santa Susana Field Laboratory, Los Angeles, California. Partial meltdowns.
- 1965 Philippine Sea A-4 crash, where a Skyhawk attack aircraft with a nuclear weapon fell into the sea. The pilot, the aircraft, and the B43 nuclear bomb were never recovered. It was not until the 1980s that the Pentagon revealed the loss of the one-megaton bomb.
- October 1965: US CIA-led expedition abandons a nuclear-powered telemetry relay listening device on Nanda Devi
- January 17, 1966: the 1966 Palomares B-52 crash occurred when a B-52G bomber of the USAF collided with a KC-135 tanker during mid-air refuelling off the coast of Spain. The KC-135 was completely destroyed when its fuel load ignited, killing all four crew members. The B-52G broke apart, killing three of the seven crew members aboard. Of the fourMk28 type hydrogen bombs the B-52G carried, three were found on land near Almería, Spain. The non-nuclear explosives in two of the weapons detonated upon impact with the ground, resulting in the contamination of a 2-square-kilometer (490-acre) (0.78 square mile) area by radioactive plutonium. The fourth, which fell into the Mediterranean Sea, was recovered intact after a 2½-month-long search. January 21, 1968: the 1968 Thule Air Base B-52 crash involved a United States Air Force (USAF) B-52 bomber. The aircraft was carrying four hydrogen bombs when a cabin fire forced the crew to abandon the aircraft. Six crew members ejected safely, but one who did not have an ejection seat was killed while trying to bail out. The bomber crashed onto sea ice in Greenland, causing the nuclear payload to rupture and disperse, which resulted in widespread radioactive contamination.
- May 1968: Soviet submarine K-27 reactor near meltdown. 9 people died, 83 people were injured. In August 1968, the Project 667 A - Yankee class nuclear submarine K-140 was in the naval yard at Severodvinsk for repairs. On August 27, an uncontrolled increase of the reactor's power occurred following work to upgrade the vessel. One of the reactors started up automatically when the control rods were raised to a higher position. Power increased to 18 times its normal amount, while pressure and temperature levels in the reactor increased to four times the normal amount. The automatic start-up of the reactor was caused by the incorrect installation of the control rod electrical cables and by operator error. Radiation levels aboard the vessel deteriorated.
- 10/12/1968 - Mayak, Former Soviet Union. Criticality accident. Plutonium solution was poured into a cylindrical container with dangerous geometry. One person died, another took a high dose of radiation and radiation sickness, after which he had two legs and his right arm amputated.
- January 1969: Lucens reactor in Switzerland undergoes partial core meltdown leading to massive radioactive contamination of a cavern. 1970s
- 1974–1976: Columbus radiotherapy accident, 10 fatalities, 88 injuries from Cobalt-60 source. July 1978: Anatoli Bugorski was working on U-70, the largest Soviet particle accelerator, when he accidentally exposed his head directly to the proton beam. He survived, despite suffering some long-term damage.
- July 1979: Church Rock Uranium Mill Spill in New Mexico, USA, when United Nuclear Corporation's uranium mill tailings disposal pond breached its dam. Over 1,000 tons of radioactive mill waste and millions of gallons of mine effluent flowed into the Puerco River, and contaminants traveled downstream. 1980s
- 1980: Houston radiotherapy accident, 7 fatalities.
- October 5, 1982: Lost radiation source, Baku, Azerbaijan, USSR. 5 fatalities, 13 injuries.
- March 1984: Radiation accident in Morocco, eight fatalities from overexposure to radiation from a lost iridium-192source.
- 1984: Fernald Feed Materials Production Center gained notoriety when it was learned that the plant was releasing millions of pounds of uranium dust into the atmosphere, causing major radioactive contamination of the surrounding areas. That same year, employee Dave Bocks, a 39 year old pipefitter, disappeared during the facility's graveyard shift and was later reported missing. Eventually, his remains were discovered inside a uranium processing furnace located in Plant 6.
- August 1985: Soviet submarine K-431 accident. Ten fatalities and 49 other people suffered radiation injuries.
- October 1986: Soviet submarine K-219 reactor almost had a meltdown. Sergei Preminin died after he manually lowered the control rods, and stopped the explosion. The submarine sank three days later.
- September 1987: Goiania accident. Four fatalities, and following radiological screening of more than 100,000 people, it was ascertained that 249 people received serious radiation contamination from exposure to Cesium-137. In the cleanup operation, topsoil had to be removed from several sites, and several houses were demolished. All the

objects from within those houses were removed and examined. *Time* magazine has identified the accident as one of the world's "worstnuclear disasters" and the International Atomic Energy Agency called it "one of the world's worst radiological incidents".

- 1989: San Salvador, El Salvador; one fatality due to violation of safety rules at Cobalt-60 irradiation facility. 1990s
- 1990: Soreq, Israel; one fatality due to violation of safety rules at Cobalt-60 irradiation facility. December 16 - 1990: radiotherapy accident in Zaragoza. Eleven fatalities and 27 other patients were injured.
- 1991: Neswizh, Belarus; one fatality due to violation of safety rules at Cobalt-60 irradiation facility.
- 1992: Jilin, China; three fatalities at Cobalt-60 irradiation facility.
- 1992: USA; one fatality.
- April 1993: accident at the Tomsk-7 Reprocessing Complex, when a tank exploded while being cleaned with nitric acid. The explosion released a cloud of radioactive gas. (INES level 4).
- 1994: Tammiku, Estonia; one fatality from disposed caesium-137 source.
- August — December 1996: Radiotherapy accident in Costa Rica. Thirteen fatalities and 114 other patients received an overdose of radiation.
- 1996: an accident at Pelindaba research facility in South Africa results in the exposure of workers to radiation. Harold Daniels and several others die from cancers and radiation burns related to the exposure.
- June 1997: Sarov, Russia; one fatality due to violation of safety rules.
- May 1998: The Acerinox accident was an incident of radioactive contamination in Southern Spain. A caesium-137 source managed to pass through the monitoring equipment in an Acerinox scrap metal reprocessing plant. When melted, the caesium-137 caused the release of a radioactive cloud.
- September 1999: two fatalities at criticality accident at Tokaimura nuclear accident (Japan) 2000s
- January–February 2000: Samut Prakan radiation accident: three deaths and ten injuries resulted in Samut Prakarn when acobalt-60 radiation-therapy unit was dismantled.
- May 2000: Meet Halfa, Egypt; two fatalities due to radiography accident.
- August 2000 – March 2001: Instituto Oncologico Nacional of Panama, 17 fatalities. Patients receiving treatment for prostate cancer and cancer of the cervix receive lethal doses of radiation.
- August 9, 2004: Mihama Nuclear Power Plant accident, 4 fatalities. Hot water and steam leaked from a broken pipe (not actually a radiation accident).
- 9 May 2005: it was announced that Thermal Oxide Reprocessing Plant in the UK suffered a large leak of a highly radioactive solution, which first started in July 2004.
- April 2010: Mayapuri radiological accident, India, one fatality after a cobalt-60 research irradiator was sold to a scrap metal dealer and dismantled.
- 2010s
- March 2011: Fukushima I nuclear accidents, Japan and the radioactive discharge at the Fukushima Daiichi Power Station.
- January 17, 2014: At the Rössing Uranium Mine, Namibia, a catastrophic structural failure of a leach tank resulted in a major spill. The France-based laboratory, CRIIAD, reported elevated levels of radioactive materials in the area surrounding the mine. Workers were not informed of the dangers of working with radioactive materials and the health effects thereof.
- February 1, 2014: Designed to last tens thousand years, the Waste Isolation Pilot Plant (WIPP) site had its first leak of airborne radioactive materials. 140 employees working underground at the time were sheltered indoors. 13 of these tested positive for internal radioactive contamination. Internal exposure to radioactive isotopes is more serious than external exposure, as these particles lodge in the body for decades, irradiating the surrounding tissues, thus increasing the risk of future cancers and other health effects. A second leak at the plant occurred shortly after the first, releasing plutonium and other radiotoxins, causing concern for communities living near the repository.

Stochastic behavior of systems operating under changing environments has widely been studied. . Dhillon , B.S. and Natesan, J. (1983) studied an outdoor power systems in fluctuating environment . Kan Cheng (1985) has studied reliability analysis of a system in a randomly changing environment. Jinhua Cao (1989) has studied a man machine system operating under changing environment subject to a Markov process with two states. The change in operating conditions viz. fluctuations of voltage, corrosive atmosphere, very low gravity etc. may make a system completely inoperative. Severe environmental conditions can make the actual mission duration longer than the ideal mission duration. In this paper we have taken failure due to radioactivity or Overheating, steam explosion, fire, and meltdown.

When the main operative unit fails then cold standby system becomes operative. Overheating, steam explosion, fire, and meltdown failure cannot occur simultaneously in both the units and after failure the unit undergoes repair facility of very high cost in case of Overheating, steam explosion, fire, and meltdown immediately. The repair is done on the basis of first fail first repaired.

Assumptions

1. $\lambda_1, \lambda_2,$ are constant failure rates for failure due to radioactivity or Overheating, steam explosion, fire, and meltdown respectively. The CDF of repair time distribution of Type I and Type II are $G_1(t)$ and $G_2(t)$.
2. The failure due to Overheating, steam explosion, fire, and meltdown is non-instantaneous and it cannot come simultaneously in both the units.
3. The repair starts immediately after the failure due to radioactivity or Overheating, steam explosion, fire, and meltdown works on the principle of first fail first repaired basis.
4. The repair facility does no damage to the units and after repair units are as good as new.
5. The switches are perfect and instantaneous.
6. All random variables are mutually independent.
7. When both the units fail, we give priority to operative unit for repair.
8. Repairs are perfect and failure of a unit is detected immediately and perfectly.
9. The system is down when both the units are non-operative.

Notations

λ_1 , λ_2 are the failure rates due to radioactivity or Overheating, steam explosion, fire, and meltdown respectively. $G_1(t)$, $G_2(t)$ – repair time distribution Type -I, Type-II due to radioactivity or Overheating, steam explosion, fire, and meltdown respectively.

p, q - probability of failure due to radioactivity ; Overheating, steam explosion, fire, and meltdown respectively such that p+ q=1

$M_i(t)$ System having started from state i is up at time t without visiting any other regenerative state

A_i (t) state is up state as instant t

R_i (t) System having started from state i is busy for repair at time t without visiting any other regenerative state.

B_i (t) the server is busy for repair at time t.

$H_i(t)$ Expected number of visits by the server for repairing given that the system initially starts from regenerative state i

Symbols for states of the System

Superscripts O, CS, RAF, OSFMF

Operative, Cold Standby, failure due to radioactivity or Failure due to Overheating, steam explosion, fire, and meltdown respectively

Subscripts nosfm, osfm, raf, ur, wr, uR

No Overheating, steam explosion, fire, and meltdown, Overheating, steam explosion, fire, and meltdown, failure due to radioactivity, under repair, waiting for repair, under repair continued from previous state respectively

Up states – 0, 1, 2, 7, 8 ;

Down states – 3, 4, 5, 6

regeneration point – 0,1,2, 7, 8

States of the System

$0(O_{nosfm}, CS_{nosfm})$

One unit is operative and the other unit is cold standby and there is no failure due to Overheating, steam explosion, fire, and meltdown in both the units.

$1(RAF_{raf, ur}, O_{nosfm})$

The operating unit fails due to radioactivity and is under repair immediately of very costly Type- I and standby unit starts operating with no failure due to Overheating, steam explosion, fire, and meltdown.

$2(OSFMF_{osfm, ur}, O_{nosfm})$

The operative unit fails due to OSFMF resulting from failure due to Overheating, steam explosion, fire, and meltdown and undergoes repair of type II and the standby unit becomes operative with no failure due to Overheating, steam explosion, fire, and meltdown.

$3(RAF_{raf,uR}, OSFMF_{osfm,wr})$

The first unit fails due to radioactivity and under very costly Type-1repair is continued from state 1 and the other unit fails due to OSFMF resulting from Failure due to Overheating, steam explosion, fire, and meltdown and is waiting for repair of Type -II.

$4(RAF_{raf,uR}, RAF_{raf,wr})$

The repair of the unit is failed due to RAF resulting from failure due to radioactivity is continued from state 1and the other unit failed due to RAF resulting from failure due to radioactivity is waiting for repair of Type-I.

$5(OSFMF_{osfm, uR}, OSFMF_{osfm, wr})$

The operating unit fails due to failure due to Overheating, steam explosion, fire, and meltdown (OSFMF mode) and under repair of Type - II continues from the state 2 and the other unit fails also due to failure due to Overheating, steam explosion, fire, and meltdown is waiting for repair of Type- II.

$6(OSFMF_{osfm,uR}, RAF_{raf,wr})$

The operative unit fails due to OSFMF resulting from failure due to Overheating, steam explosion, fire, and meltdown and under repair continues from state 2 of Type –II and the other unit is failed due to RAF resulting from failure due to radioactivity and under very costly Type-1

7(O $_{nosfm}$, RAF $_{raf,ur}$)

The repair of the unit failed due to RAF resulting from failure due to radioactivity failure is completed and there is no failure due to Overheating, steam explosion, fire, and meltdown and the other unit is failed due to RAF resulting from failure due to radioactivity is under repair of very costly Type-1

8(O $_{nosfm}$, OSFMF $_{osfm,ur}$)

The repair of the unit failed due to RAF resulting from failure due to radioactivity failure is completed and there is no failure due to Overheating, steam explosion, fire, and meltdown and the other unit is failed due to OSFMF resulting from failure due to Overheating, steam explosion, fire, and meltdown is under repair of Type-II.

Transition Probabilities

Simple probabilistic considerations yield the following expressions:

$p_{01} = p,\quad p_{02} = q,$

$p_{10} = pG_1^*(\lambda_1)+q\,G_1^*(\lambda_2)= p_{70}\,,$

$p_{20} = pG_2^*(\lambda_1)+q\,G_2^*(\lambda_2)= p_{80}\,,$

$p_{11}^{(3)}= p(1- G_1^*(\lambda_1))= p_{14} = p_{71}^{(4)}\; p_{28}^{(5)}= q(1- G_2^*(\lambda_2))= p_{25} = p_{82}^{(5)}$ (1)

We can easily verify that

$p_{01} + p_{02} = 1,\; p_{10} + p_{17}^{(4)} (= p_{14}) + p_{18}^{(3)} (=p_{13}) = 1,$

$p_{80} + p_{82}^{(5)} + p_{87}^{(6)} = 1$ (2)

And mean sojourn time is

$\mu_0 = E(T) = \int_0^\infty P[T > t]dt$

Mean Time To System Failure

$Ø_0(t) = Q_{01}(t)[\bar{s}]\, Ø_1(t) + Q_{02}(t)[s]\, Ø_2(t)$

$Ø_1(t) = Q_{10}(t)[s]\, Ø_0(t) + Q_{13}(t) + Q_{14}(t)$

$Ø_2(t) = Q_{20}(t)[s]\, Ø_0(t) + Q_{25}(t) + Q_{26}(t)$ (3-5)

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-5) and solving for

$ø_0^*(s) = N_1(s) / D_1(s)$ (6)

where

$N_1(s) = Q_{01}^*[Q_{13}^*(s) + Q_{14}^*(s)] + Q_{02}^*[Q_{25}^*(s) + Q_{26}^*(s)]$

$D_1(s) = 1 - Q_{01}^*\, Q_{10}^* - Q_{02}^*\, Q_{20}^*$

Making use of relations (1) & (2) it can be shown that $ø_0^*(0) =1$, which implies that $ø_0(t)$ is a proper distribution.

$$\text{MTSF} = E[T] = \left.\frac{d}{ds}\, \bar{ø}_0(s)\right|_{s=0}$$

$= (D_1'(0) - N_1'(0)) / D_1(0)$

$= (\mu_0 + p_{01}\,\mu_1 + p_{02}\,\mu_2) / (1 - p_{01}\, p_{10} - p_{02}\, p_{20})$

where

$\mu_0 = \mu_{01} + \mu_{02}\,,$

$\mu_1 = \mu_{01} + \mu_{17}^{(4)} + \mu_{18}^{(3)},$

$\mu_2 = \mu_{02} + \mu_{27}^{(6)} + \mu_{28}^{(5)}$

Availability analysis

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have

$M_0(t) = e^{-\lambda_1 t} e^{-\lambda_2 t}\,,$

$M_1(t) = p\,\overline{G_1(t)}\, e^{-(\lambda_1+\lambda_2)} = M_7(t)$

$M_2(t) = q\,\overline{G_2(t)}\, e^{-(\lambda_1+\lambda_2)} = M_8(t)$

The point wise availability $A_i(t)$ have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t)$

$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{18}^{(3)}(t)[c]A_8(t) + q_{17}^{(4)}(t)[c]A_7(t)$

$A_2(t) = M_2(t) + q_{20}(t)[c]A_0(t) + [q_{28}^{(5)}(t)[c]\, A_8(t) + q_{27}^{(6)}(t)]\, [c]A_7(t)$

$A_7(t) = M_7(t) + q_{70}(t)[c]A_0(t) + [q_{71}^{(4)}(t)[c]\, A_1(t) + q_{78}^{(3)}(t)]\, [c]A_8(t)$ $A_8(t) = M_8(t) + q_{80}(t)[c]A_0(t) +[q_{82}^{(5)}(t)[c]\, A_2(t) + q_{87}^{(6)}(t)]\, [c]A_7(t)$ (7-11)

Taking Laplace Transform of eq. (7-11) and solving for $\hat{A}_0(s)$

$\hat{A}_0(s) = N_2(s) / D_2(s)$ (12)

where

$N_2(s) = \hat{M}_0 (1 - \hat{q}_{78}^{(3)} - \hat{q}_{87}^{(6)}) - \hat{q}_{82}^{(5)}$

$(\hat{q}_{27}^{(6)}\, \hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)} - \hat{q}_{71}^{(4)}$

$(\hat{q}_{17}^{(4)} + \hat{q}_{87}^{(6)}\, \hat{q}_{18}^{(3)}) + \hat{q}_{71}^{(4)}\, \hat{q}_{82}^{(5)}$

$(\hat{q}_{17}^{(4)} - \hat{q}_{27}^{(6)}\, \hat{q}_{18}^{(3)})] + \hat{q}_{01}[\hat{M}_1(1 -$

$\hat{q}_{78}^{(3)}\, \hat{q}_{87}^{(6)}) + \hat{q}_{71}^{(4)}(\hat{M}_7 + \hat{q}_{78}^{(3)}$

$\hat{M}_8) + \hat{q}_{18}^{(3)}(\hat{M}_7\, \hat{q}_{87}^{(6)} - \hat{M}_8) -$

$\hat{q}_{82}^{(5)}(\tilde{M}_1(\hat{q}_{27}^{(6)}\hat{q}_{78}^{(3)}+\hat{q}_{28}^{(5)})+$

$\hat{q}_{17}^{(4)}(-\tilde{M}_2(\hat{q}_{78}^{(3)}+\tilde{M}_7\hat{q}_{28}^{(5)})-$

$\hat{q}_{18}^{(3)}(\tilde{M}_2+\tilde{M}_7\hat{q}_{27}^{(6)})\}]\ \hat{q}_{02}[\tilde{M}_2(1-\hat{q}_{78}^{(3)}\ \hat{q}_{87}^{(6)})+\hat{q}_{27}^{(6)}($

$\tilde{M}_7+\hat{q}_{78}^{(3)}\tilde{M}_8)+\hat{q}_{28}^{(5)}(\tilde{M}_7\hat{q}_{87}^{(6)}+\tilde{M}_8)-\hat{q}_{71}^{(4)}(\tilde{M}_1(-\hat{q}_{27}^{(6)}-\hat{q}_{28}^{(5)}+$

$\hat{q}_{87}^{(6)})+\hat{q}_{17}^{(4)}(\tilde{M}_2+\hat{q}_{28}^{(5)}\tilde{M}_8)-\hat{q}_{18}^{(3)}(-\tilde{M}_2\ \hat{q}_{87}^{(6)}+\tilde{M}_8\ \hat{q}_{27}^{(6)})\}]$

$\hat{q}_{18}^{(3)}(\tilde{M}_2+\tilde{M}_7\hat{q}_{27}^{(6)})\}]$

$D_2(s)=(1-\hat{q}_{78}^{(3)}-\hat{q}_{87}^{(6)})-\hat{q}_{82}^{(5)}($

$\hat{q}_{27}^{(6)}\ \hat{q}_{78}^{(3)}+\hat{q}_{28}^{(5)})-\hat{q}_{71}^{(4)}$

$(\hat{q}_{17}^{(4)}+\hat{q}_{87}^{(6)}\ \hat{q}_{18}^{(3)})+\hat{q}_{71}^{(4)}\ \hat{q}_{82}^{(5)}(\hat{q}_{17}^{(4)}\ \hat{q}_{28}^{(5)}-\hat{q}_{18}^{(3)})]+\hat{q}_{01}[-\hat{q}_{10}(1-$

$\hat{q}_{78}^{(3)}\ \hat{q}_{87}^{(6)})-\hat{q}_{71}^{(4)}(\hat{q}_{70}+\hat{q}_{78}^{(3)}$

$\hat{q}_{80})-\hat{q}_{18}^{(3)}(\hat{q}_{70}\ \hat{q}_{87}^{(6)}-\hat{q}_{80})-$

$\hat{q}_{82}^{(5)}(-\hat{q}_{10}(\hat{q}_{27}^{(6)}\ \hat{q}_{78}^{(3)}+\hat{q}_{28}^{(5)})+$

$\hat{q}_{17}^{(4)}(\hat{q}_{20}(\hat{q}_{78}^{(3)}-\hat{q}_{70}\hat{q}_{28}^{(5)})+$

$\hat{q}_{18}^{(3)}(\hat{q}_{20}+\hat{q}_{70}\hat{q}_{27}^{(6)})\}]\ \hat{q}_{02}[-\hat{q}_{20}(1-\hat{q}_{78}^{(3)}\ \hat{q}_{87}^{(6)})-\hat{q}_{27}^{(6)}(\hat{q}_{70}+\hat{q}_{78}^{(3)}\ \hat{q}_{80})-\hat{q}_{28}^{(5)}(\hat{q}_{70}\hat{q}_{87}^{(6)}+\hat{q}_{80})-\hat{q}_{71}^{(4)}($

$\hat{q}_{10}(\hat{q}_{27}^{(6)}+\hat{q}_{28}^{(5)}\ \hat{q}_{87}^{(6)})-\hat{q}_{17}^{(4)}(\hat{q}_{20}-$

$\hat{q}_{28}^{(5)}\ \hat{q}_{80})-\hat{q}_{18}^{(3)}(\hat{q}_{20}\ \hat{q}_{87}^{(6)}+\hat{q}_{80}$

$\hat{q}_{27}^{(6)})\}]$

(Omitting the arguments s for brevity)

The steady state availability

$A_0=\lim_{t\to\infty}[A_0(t)]$

$=\lim_{s\to 0}[s\hat{A}_0(s)]\ =\lim_{s\to 0}\frac{s\,N_2(s)}{D_2(s)}$

Using L' Hospitals rule, we get

$A_0=\lim_{s\to 0}\frac{N_2(s)+s\ N_2'(s)}{D_2'(s)}=\frac{N_2(0)}{D_2'(0)}$ (13)

The expected up time of the system in (0,t] is

$\lambda_u(t)=\int_0^\infty A_0(z)dz$

So that $\bar{\lambda}_u(s)=\frac{\hat{A}_0(s)}{s}=\frac{N_2(S)}{SD_2(S)}$ (14)

The expected down time of the system in (0,t] is

$\lambda_d(t)=t-\lambda_u(t)$

So that $\bar{\lambda}_d(s)=\frac{1}{s^2}-\bar{\lambda}_u(s)$ (15)

The expected busy period of the server when there is OSFMF - Failure due to Overheating, steam explosion, fire, and meltdown or RAF- failure due to Failure due to radioactivity in (0,t]

$R_0(t)=q_{01}(t)[c]R_1(t)+q_{02}(t)[c]R_2(t)$

$R_1(t)=S_1(t)+q_{10}(t)[c]R_0(t)+$
$q_{18}^{(3)}(t)[c]R_8(t)+q_{17}^{(4)}(t)[c]R_7(t)$

$R_2(t)=S_2(t)+q_{20}(t)[c]R_0(t)+q_{28}^{(5)}(t)$
$R_8(t)+q_{27}^{(6)}(t)][c]R_7(t)$

$R_7(t)=S_7(t)+q_{70}(t)[c]R_0(t)+Q_{71}^{(4)}(t)$
$R_1(t)+q_{78}^{(3)}(t)][c]R_8(t)$

$R_8(t)=S_8(t)+q_{80}(t)[c]R_0(t)+Q_{82}^{(5)}(t)$
$R_2(t)+q_{87}^{(6)}(t)][c]R_7(t)$ (16-20)

Taking Laplace Transform of eq. (16-20) and solving for $\bar{R}_0(s)$

$\bar{R}_0(s)=N_3(s)\ /\ D_2(s)$ (21)

where

$N_3(s)=\hat{q}_{01}[\hat{S}_1(1-\hat{q}_{78}^{(3)}\ \hat{q}_{87}^{(6)})+$

$\hat{q}_{71}^{(4)}(\hat{S}_7+\hat{q}_{78}^{(3)}\hat{S}_8)+\hat{q}_{18}^{(3)}(\hat{S}_7$

$\hat{q}_{87}^{(6)}-\hat{S}_8)]-\hat{q}_{01}\hat{q}_{82}^{(5)}(\hat{S}_1\ \hat{q}_{27}^{(6)}\ \hat{q}_{78}^{(3)}+\hat{q}_{28}^{(5)})+\hat{q}_{17}^{(4)}(\hat{S}_2\ \hat{q}_{78}^{(3)}+\hat{S}_7\hat{q}_{28}^{(5)})-$

$\hat{q}_{18}^{(3)}(\hat{S}_2+\hat{S}_7\hat{q}_{27}^{(6)})]+\hat{q}_{02}[\hat{S}_2(1-$

$\hat{q}_{78}^{(3)}\ \hat{q}_{87}^{(6)})+\hat{q}_{27}^{(6)}(\hat{S}_7+\hat{q}_{78}^{(3)}\hat{S}_8)+\hat{q}_{28}^{(5)}(\hat{S}_7\ \hat{q}_{87}^{(6)}+\hat{S}_8)-\hat{q}_{02}\hat{q}_{71}^{(4)}(\hat{S}_1(-$

$\hat{q}_{27}^{(6)}-\hat{q}_{28}^{(5)}\ \hat{q}_{87}^{(6)}\ \hat{q}_{17}^{(4)}(\hat{S}_2+\hat{q}_{28}^{(5)}\hat{S}_8)-\hat{q}_{18}^{(3)}(-\hat{S}_2\ \hat{q}_{87}^{(6)}+\hat{S}_8\ \hat{q}_{27}^{(6)})]$

and

$D_2(s)$ is already defined.

(Omitting the arguments s for brevity)

In the long run, $R_0=\frac{N_3(0)}{D_2'(0)}$ (22)

The expected period of the system under OSFMF - failure due to Overheating, steam explosion, fire, and meltdown or RAF- Failure due to radioactivity in (0,t] is

$\lambda_{rv}(t)=\int_0^\infty R_0(z)dz$ So that $\bar{\lambda}_{rv}(s)=\frac{\hat{R}_0(s)}{s}$

The expected number of visits by the repairman for repairing the identical units in (0,t]

$H_0(t)=Q_{01}(t)[s][1+H_1(t)]\ +$

$Q_{02}(t)[s][1+ H_2(t)]$

$H_1(t) = Q_{10}(t)[s]H_0(t)] + Q_{18}^{(3)}(t)[s]$

$H_8(t) + Q_{17}^{(4)}(t)] [s]H_7(t)$,

$H_2(t) = Q_{20}(t)[s]H_0(t) + Q_{28}^{(5)}(t) [s]$

$H_8(t) +Q_{27}^{(6)}(t)] [c]H_7(t)$

$H_7(t) = Q_{70}(t)[s]H_0(t) + Q_{71}^{(4)}(t) [s]$

$H_1(t) +Q_{78}^{(3)}(t)] [c]H_8(t)$

$H_8(t) = Q_{80}(t)[s]H_0(t) + Q_{82}^{(5)}(t) [s]$

$H_2(t) +Q_{87}^{(6)}(t)] [c]H_7(t)$ (23-27)

Taking Laplace Transform of eq. (23-27) and solving for $H_0^*(s)$

$$H_0^*(s) = N_4(s) / D_3(s) \qquad (28)$$

In the long run , $H_0 = N_4(0) / D_3'(0)$ (29)

Benefit- Function Analysis

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure due to radioactivity or Failure due to Overheating, steam explosion, fire, and meltdown, expected number of visits by the repairman for unit failure.

The expected total Benefit-Function incurred in (0,t] is

C(t) = Expected total revenue in (0,t]

- expected busy period of the system under failure due to radioactivity or failure due to Overheating, steam explosion, fire, and meltdown for repairing the units in (0,t]

- expected number of visits by the repairman for repairing of identical the units in (0,t]

The expected total cost per unit time in steady state is

$$C = \lim_{t\to\infty}(C(t)/t) = \lim_{s\to 0}(s^2C(s))$$

$$= K_1A_0 - K_2R_0 - K_3H_0$$

where

K_1 - revenue per unit up-time,

K_2 - cost per unit time for which the system is under repair of type- I or type- II

K_3 - cost per visit by the repairman for units repair.

CONCLUSION

After studying the system , we have analyzed graphically that when the failure rate due to radioactivity or Failure due to Overheating, steam explosion, fire, and meltdown increases, the MTSF and steady state availability decreases and the Benefit-function decreased as the failure increases.

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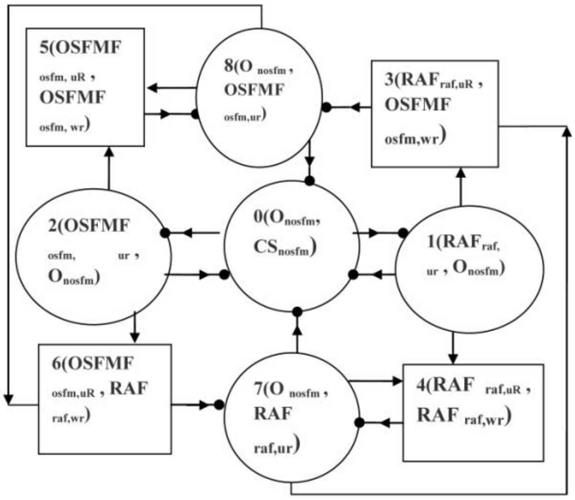

Fig. The State Space Diagram

○Up state □ down state

● Regeration point

◆ ◆ ◆