A Case Study: Comparison of Newton-Raphson and Gauss-Seidal Load Flow Solution Techniques in Distributed Transmission and Generation Electricity Networks

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Abstract : Many methods have been proposed to solve the problem of power flow in power systems, since the first formulation made in the 1950s. Among these methods, the iterative methods are widely used. From these methods, Newton–Raphson power flow method is most preferred and has also become a reference point for calculating power flow due to a fast present and efficient convergence. Along with this, Gauss-Seidal is also used for load flow solutions. In this paper, we have presented the basic mechanism of these iterative methods and a case study has been carried using IEEE 30, IEEE 57 and IEEE118 bus systems. Load flow analysis is used to compute precise steady-state voltages magnitudes and angles of all buses in the network, the real and reactive power flows into every line and transformer, under the assumption of known generation and load. Model of power system generates non-linear algebraic equations and to solve these equations, two solution algorithms based on power equations of the methods were adopted. Experiment results shows that number of iterations increases proportionally with the number of buses for Gauss-Seidel technique while that of Newton–Raphson method remained almost practically constant even with varying number of buses. Also Gauss-Seidal is best suited for systems have small number of buses and do not converge on large bus systems.

Keywords –Load flow analysis, Newton-Raphson, Gauss-Seidal, IEEE 30, 57,118 bus systems

I. INTRODUCTION

The studies for the load flow calculations started with the Ward & Hale method in 1956 [1], and currently the Newton-Raphson (NR) method are being widely used. Load flow analysis enables the planning of the future growth of power systems in the most economical way and determining the best operation of existing systems. Successful power system operation under normal balanced threephase steady-state conditions necessitates that generation supplies demand and losses, bus voltage magnitudes remain nearly at rated values, generators operate within rated real and reactive power limits, and transmission lines and transformers are not overloaded. The load flow or power flow computer program is the basic tool for investigating these requirements. To perform the load flow analysis, the operating conditions must always be selected for each study. The assumption that the system is operating under balanced conditions is made and a single-phase model is used [2]. The system buses are classified into three types. At each bus except one, the net real power into the network must be specified. In addition, at these buses either the net flow of reactive power into the network or the magnitude of the voltage must be specified as a constant. A swing bus, generally connected to the generator bus, is left without specified real power flow [3]. The Gauss-Seidel method was widely used before the introduction of Newton-Raphson method (NR) which is more reliable but has higher memory requirements. The method was introduced in 1961 and has been made practical by using optimally ordered Gaussian elimination and special programming techniques [4]. The use of sparsity programming and optimal ordering has reduced memory requirements and increased the popularity of the method [5]. Performance of the conventional NR method has continued to receive attention in recent years. In 2009, Yubin et al. proposed a method of constructing the Jacobian matrix in Newton power flow expressed in rectangular form. The small impedance branch may result in the non-convergence of Newton power flow method in both rectangular form and polar form. Analysis showed the configuration of the Jacobian matrix plays a big part in the convergence of Newton power flow in rectangular form particularly when it is applied in systems with small impedance branches. Quoting the paper, the divergence problem can be avoided by modifying the Jacobian matrix [6]. Real power transmission system involves a large number of buses. Consider the eastern interconnected system, which would require over 150,000 major components in a power flowmodel [7]. Computation for such wide-area events will require large amount of time. This paper responds to the issue by presenting a brief survey of existed algorithms for load flow analysis.

II. LOAD FLOW ANALYSIS

The load flow analysis, in power system parlance, is the steady state solution of the power system network. The power flow problem involves determining voltages and line flows, in a large sparse electrical network, for a given load and generation schedule [8]. The power system network can be modeled as an electric network

and solved for the steady state power, voltages at various buses and hence the power at the slack bus and power flows through inter connecting power channels [9]. Four quantities which are associated with each bus are voltage magnitude |v|, phase angle δ, real power P, and reactive power Q. The system buses are generally classified into three types namely: Slack bus, Load buses and Generator buses where two variables are specified and others two to be determined [10].

III. POWER FLOW EQUATIONS

Steady-state analysis of an interconnected power system during normal operation can be performed via the extrapolation of the theory presented above in a system known as the power flow equations. For the most part, the analysis assumes that the network is balanced and can therefore be represented by a single-phase (or one-line) network. The network equations can be formulated systematically in a variety of forms; however the most common approach is the node-voltage method as detailed in [8]. The steady-state solution of an AC electrical network is governed by the matrix equation:

$$\bar{I} = \overline{YV} \text{.....} 1$$

Where: I is an n-dimensional vector of network currents injected on each system bus; V is an n-dimensional vector of system voltages at each system bus; and, Y is an n x n matrix of network admittance; which is a convenient representation of the inverted network complex impedances. The diagonal elements of the admittance matrix are the sum of all the admittances connected to each node, known as the self-admittance:

$$Y_{ii} = \sum_{j=0}^{n} Y_{ij} \text{.....} 2$$

$$where \quad j \neq i$$

The off-diagonal elements are equal to the negative of the mutual admittance between network nodes.

$$Y_{ij} = Y_{ji} = -y_{ij} \text{.....} 3$$

To demonstrate, consider a typical system busbar, i, connected to n others as shown in Figure 1.

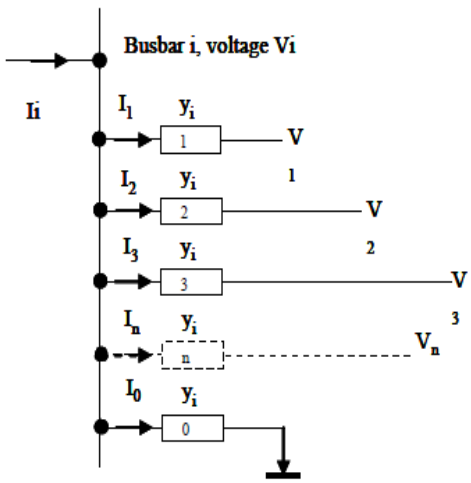

Figure 1: A typical electrical power system bus

The network bus bars are all connected via the intervening impedance of each respective circuit branch, Zi1, Zi2, Zi3 … Zin. The system impedances are inverted to form the system admittances, yi1, yi2, yi3 … yin.

Application of Kirchhoff's Current Law (KCL) then yields:

$$I_i = y_{io}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + \text{.....} y_{in}(V_i - V_n)$$
$$= (y_{io} + y_{i1} + y_{i2} + \text{.....} + y_{in})V_i - y_{i1}V_1 - y_{i2}V_2 - y_{i3}V_3 - \text{.....} - y_{in}V_n$$
$$I_i = V_i \sum_{j=0}^{n} y_{ij} - V_j \sum_{j=1}^{n} y_{ij} \text{.....} 4$$
$$where \quad j \neq i$$

The formulation of the network equations in the nodal admittance form results in complex linear simultaneous equations in terms of node currents. However, in a power system it is typically the system power injections and absorptions that are known, not the currents.

Given the real and reactive power injections at bus i are:

$$P_i + jQ_i = V_i I_i^* \ or \ I_i = \frac{P_i - jQ_i}{V_i^*}$$

Substituting for Ii in (12) gives:

$$\frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^{n} yij - V_j \sum_{j=1}^{n} y_{ij} \quad j \neq i$$

Therefore the resulting mathematical formulation, in terms of power, is a system of nonlinear equations, which can only be solved by iterative means subject to the nodal power balance criteria.

After an iterative solution of the system node voltages, network line currents to and from each system node are calculated via (4). The real power losses and reactive power charge in each network line and transformer are then deducted from the sum of the determined power flows. Where the complex power ij S from bus i to j and ji S from bus j to i are:

$$\text{S } S_{ij} = V_i I_{ij}^i \ \& \ S_{ji} = V_j I_{ji}^* \text{.....} 5$$

The power loss in line i-j is the algebraic sum of the power flows:

$$S_{Lij} = S_{ij} + S_{ji} \text{.....} 6$$

For larger power systems networks, particularly for high resolution time sequential solutions, this is only feasible by computational methods.

IV. PRESENTED METHODS

1.1 Gauss-Seidel Method

The Gauss-Seidel method is an iterative algorithm for solving a set of non-linear algebraic equations. It was one of the methods used in load flow studies. Here a solution vector is assumed and one of the equations is used to obtain the revised value of a particular variable.

The solution vector is immediately updated in respect of this variable. The process is then repeated for all the variables thereby completing one-iteration. The iterative process is then repeated till the solution vector converges within prescribed accuracy. The convergence is quite sensitive to the starting values assumed [11]. The advantages of the method are the simplicity of the technique, small computer memory requirement, less computational time per iteration. However the disadvantages are slow rate of convergence, large numbers of iterations, increase of numbers of iteration directly with the increase in the number of buses and effect of convergence due to choice of slack bus. In view of these disadvantages, Gauss-Seidel method is used only for system having small number of buses [12]. An initial solution vector is assumed, chosen from past experiences, statistical data or from practical considerations. At every subsequent iteration, the solution is updated till convergence is reached. The GS method applied to power flow problem is as discussed below.

Case (a): Systems with PQ buses only:

Initially assume all buses to be PQ type buses, except the slack bus. This means that (n–1) complex bus voltages have to be determined. For ease of programming, the slack bus is generally numbered as bus-1. PV buses are numbered in sequence and PQ buses are ordered next in sequence. This makes programming easier, compared to random ordering of buses. Consider the expression for the complex power at bus-i, given as:

$$S_i = V_i \left(\sum_{j=1}^{n} Y_{ij} V_j \right)^* \text{.....(7)}$$

This can be written as

$$S_i^* = V_i^* \left(\sum_{j=1}^{n} Y_{ij} V_j \right) \text{.....(8)}$$

Since *

$S_i^* = Pi - jQi, we\ get$ we get,

$$\frac{Pi - jQi}{V_i^*} = \sum_{j=1}^{n} Y_{ij} V_j$$

$so\ that$

$$\frac{Pi - jQi}{V_i^*} = Y_{ii} V_i + \sum_{j-1 \& j \neq i}^{n} Y_{ij} V_j \text{.....(9)}$$

Rearranging the terms, we get,

$$V_i = \frac{1}{Y_{ii}} \left[\frac{Pi - jQi}{V_i^*} - \sum_{j-1 \& j \neq i}^{n} Y_{ij} V_j \right] \text{.....(10)}$$

$\forall$ i = 2, 3……..n

Equation (10) is an implicit equation since the unknown variable, appears on both sides of the equation. Hence, it needs to be solved by an iterative technique. Starting from an initial estimate of all bus voltages, in the RHS of (10) the most recent values of the bus voltages is substituted. One iteration of the method involves computation of all the bus voltages. In Gauss–Seidel method, the value of the updated voltages is used in the computation of subsequent voltages in the same iteration, thus speeding up convergence. Iterations are carried out till the magnitudes of all bus voltages do not change by more than the tolerance value.

Algorithm for GS method

1. Prepare data for the given system as required.

2. Formulate the bus admittance matrix YBUS. This is generally done by the rule of inspection.

3. Assume initial voltages for all buses, 2,3,…n. In practical power systems, the magnitude of the bus voltages is close to 1.0 p.u. Hence, the complex bus voltages at all (n-1) buses (except slack bus) are taken to be $1.0 \angle 0^0$.

This is normally referred as the **flat start** solution.

4. Update the voltages. In any $(k+1)^{st}$ iteration, from (17) the voltages are

given by

$$V_i^{k+1} = \frac{1}{Y_{ii}} \left[\frac{Pi - jQi}{\left(V_i^k\right)^*} - \sum_{j=1}^{i-1} Y_{ij} V_j^{k+1} - \sum_{j=i+1}^{n} Y_{ij} V_j^k \right] \forall$$

i=2,3,…n **(11)**

Here note that when computation is carried out for bus-i, updated values are already available for buses 2,3….(i-1) in the current $(k+1)^{st}$iteration. Hence these values are used. For buses (i+1)…..n, values from previous, kth iteration are used.

5. Continue iterations till

$$|\Delta V_i^{k+1}| = |V_i^{k+1} - V_i^k| < \varepsilon$$

$\forall$ i = 2,3,…n…… **(12)**

Where ε is the tolerance value. Generally it is customary to use a value of 0.0001 pu.

6. Compute slack bus power after voltages have converged using (15)

[assuming bus 1 is slack bus].

$$S_1^* = P_1 - jQ_1 = V_1^* \left(\sum_{j=1}^{n} Y_{1j} V_j \right) \text{.....(13)}$$

7. Compute all line flows.

8. The complex power loss in the line is given by Sik + Ski. The total loss in the system is calculated by summing the loss over all the lines.

Case (b): Systems with PV buses also present:

At PV buses, the magnitude of voltage and not the reactive power is specified. Hence it is needed to first make an estimate of Qi to be used in (11). From (8) we have

$$Q_i = -\mathrm{Im}\left\{ V_i^* \sum_{j=1}^{n} Y_{ij} V_j \right\}$$

Where Imstands for the imaginary part. At any (k+1)stiteration, at the PV bus-i,

$$Q_i^{k+1} = -\mathrm{Im}\left[\left(V_i^k\right)^* \sum_{j=1}^{i-1} Y_{ij} V_j^{k+1} + \left(V_i^k\right)^* \sum_{j=i}^{n} Y_{ij} V_j^k \right] \text{....(15)}$$

The steps for ith PV bus are as follows:

1. Compute Q_i^{k+1} using (14)

2. Calculate Vi using (11) with Qi = Q_i^{k+1}

3. Since $|V_i|$ is specified at the PV bus, the magnitude of Vi obtained in step 2 has to be modified and set to the specified value $|V_{i,sp}|$. Therefore,

$$V_i^{k+1} = |V_{i,sp}| \angle \delta_i^{k+1} \text{.....(16)}$$

The voltage computation for PQ buses does not change.

Case (c): Systems with PV buses with reactive power generation limits specified:

In the previous algorithm if the Q limit at the voltage controlled bus is violated during any iteration, i.e. Q_i^{k+1} computed using (14) is either less than Qi, min or greater than Qi,max, it means that the voltage cannot be maintained at the specified value due to lack of reactive power support. This bus is then treated as a PQ bus in the (k+1)st iteration and the voltage is calculated with the value of Qi set as follows:

$$\begin{array}{ll} if\ Qi < Qi,min & if\ Qi > Qi,max \\ Then\ Qi = Qi,min & then\ Qi = Qi,max \end{array} \quad (17)$$

If in the subsequent iteration, if Qi falls within the limits, then the bus can be switched back to PV status.

1.2 Newton-Raphson Method

The origin of the formulation of the power flow problems and the solution based on Newton-Raphson's technique dates back to the late 1960s [13]. It is an iterative method which approximates a set of non-linear simultaneous Taylor's series expansion and the terms are limited to the first approximation [14]. Many advantages are attributed to the Newton-Raphson(NR) approach. Its convergence characteristics are relatively more powerful compared to other alternative processes and the reliability of Newton-Raphson approach is comparatively good, since it can solve cases that lead to divergence with other popular processes.Failures do occur on some ill-conditional problems [9].The NR method applied to power flow problem is as discussed below.

Newton-Raphson (NR) method is used to solve a system of non-linear algebraic equations of the form f(x) =0. Consider a set of n non-linear algebraic equations given by

$$f_i(x_1, x_2, x_3 \ldots\ldots\ldots x_n) = 0$$
$$i = 1, 2, 3 \ldots\ldots\ldots \mathrm{n} \ldots (18)$$

$Let\ x_1^0, x_2^0, x_3^0, \ldots\ldots\ldots\ldots\ldots \mathrm{x}_n^0$ be the initial guess of unknown variables and

$\square x_1^0, \square x_2^0, \square x_3^0, \ldots\ldots\ldots\ldots\ldots \square \mathrm{x}_n^0$ be the respective corrections. Therefore,

$$f_i(x_1^0 + \square x_1^0, x_2^0 + \square x_2^0, \ldots\ldots\ldots x_n^0 + \square x_n^0) = 0$$
$$i = 1, 2, 3 \ldots\ldots\ldots \mathrm{n} \ldots (19)$$

The above equation can be expanded using Taylor's series to give

$$f_i(x_1^0, x_2^0, x_3^0, \ldots\ldots\ldots\ldots\ldots \mathrm{x}_n^0) + \ldots\ldots$$

$$\ldots\left[\left(\frac{\partial f_i}{\partial x_1}\right)^0 \Delta x_1^0 + \left(\frac{\partial f_i}{\partial x_2}\right)^0 \Delta x_2^0 + \ldots\ldots + \left(\frac{\partial f_i}{\partial x_n}\right)^0 \Delta x_n^0 \right]$$

$$+ \textit{Higher order terms} = 0$$

$$\forall i = 1, 2 \ldots\ldots n \ldots\ldots (20)$$

Where,

$\left(\frac{\partial f_i}{\partial x_1}\right)^0, \left(\frac{\partial f_i}{\partial x_2}\right)^0, \ldots\ldots \left(\frac{\partial f_i}{\partial x_n}\right)^0$ are the partial derivatives of fi with respect

to $x_1 + x_2 \ldots\ldots\ldots\ldots x_n$ respectively, evaluated at $(x_1^0, x_2^0 \ldots\ldots\ldots\ldots\ldots x_n^0)$. If the higher order terms

are neglected, then (20) can be written in matrix form as

$$\begin{bmatrix} f_1^0 \\ f_2^0 \\ . \\ . \\ f_n^0 \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \cdots & \left(\frac{\partial f_1}{\partial x_1}\right)^0 \\ \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \cdots & \left(\frac{\partial f_2}{\partial x_1}\right)^0 \\ . & . & \cdots & . \\ . & . & \cdots & . \\ \left(\frac{\partial f_n}{\partial x_1}\right)^0 & \left(\frac{\partial f_n}{\partial x_1}\right)^0 & \cdots & \left(\frac{\partial f_n}{\partial x_1}\right)^0 \end{bmatrix} + \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ . \\ . \\ \Delta x_n^0 \end{bmatrix} = 0 \text{.....(21)}$$

In vector form (21) can be written as

$$F^0 + J^0 \Delta X^0 = 0$$

$$or\ F^0 = -J^0 \Delta X^0$$

$$or\ \Delta X^0 = -[J^0]^{-1} F^0 \text{....(22)}$$

$$\text{and } X^1 = X^0 + \Delta X^0 \text{.....(23)}$$

Here, the matrix [J] is called the **Jacobian** matrix. The vector of unknown variables is updated using (23). The process is continued till the difference between two successive iterations is less than the tolerance value.

NR method for load flow solution in polar coordinates

In application of the NR method, we have to first bring the equations to be solved, to the form $f_i(x_1, x_2 .....x_n) = 0\ where\ x_1, x_2 .....x_n$, the unknown variables to be determined. Let us assume that the power system has n_1 PV buses and n_2 PQ buses. In polar coordinates the unknown variables to be determined are:

(i) δ_i , the angle of the complex bus voltage at busesi, at all the PV and PQ buses. This gives us $n_1 + 2n_2$ unknown variables to be determined.

(ii) V_i , the voltage magnitude of bus i, at all the PQ buses. This gives us 2 n unknown variables to be determined. Therefore, the total number of unknown variables to be computed is: $n_1 + 2n_2$, for which we need $n_1 + 2n_2$ consistent equations to be solved. The equations are given by,

$$\Delta P_i = P_{i,sp} - P_{i,cal} = 0 \text{.....(24)}$$

$$\Delta Q_i = Q_{i,sp} - Q_{i,cal} = 0 \text{.....(25)}$$

where

$P_{i,sp}$ = Specified active power at bus i

$Q_{i,sp}$ = Specified reactive power at bus i

$P_{i,cal}$ = Calculated value of active power using voltage estimates.

$Q_{i,cal}$ = Calculated value of reactive power using voltage estimates

ΔP = Active power residue

ΔQ=Reactive power residue

The real power is specified at all the PV and PQ buses. Hence (17) is to be solved at all PV and PQ buses leading to $n_1 + 2n_2$ equations. Similarly the reactive power is specified at all the PQ buses. Hence, (18) is to be solved at all PQ buses leading to n_2 equations. We thus have $n_1 + 2n_2$ equations to be solved for $n_1 + 2n_2$ unknowns. (24) and (25) are of the form F(x) = 0. Thus NR method can be applied to solve them. Equations (24) and (25) can be written in the form of (23) as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} \text{.....(26)}$$

Where J_1, J_2, J_3, J_4 are the negated partial derivatives of ΔP and ΔQ with respect to corresponding δ and |V|. The negated partial derivative of ΔP , is same as the partial derivative of Pcal, since Psp is a constant.

Computation of Pcal and Qcal:

The real and reactive powers can be computed from the load flow equations as:

$$P_{i,cal} = P_i = \sum_{k=1}^{n} |V_1||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$= G_{ii}|V_i|^2 + \sum_{k=1\&k\neq i}^{n} |V_1||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) \text{....(27)}$$

$$Q_{i,cal} = Q_i = \sum_{k=1}^{n} |V_1||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

$$= -B_{ii}|V_i|^2 + \sum_{k=1\&k\neq i}^{n} |V_1||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) \text{....(28)}$$

The powers are computed at any $(r+1)^n$ iteration by using the voltages available from previous iteration. The

elements of the Jacobian are found using the above equations as:

ALGORITHM FOR NR

1. Formulate the YBUS

2. Assume initial voltages as follows:

0

$V_i = |V_{i,sp}| \angle 0^0$ (at all PV buses)

$V_i = 1\angle 0^0$ (at all PQ buses)

3. At $(r+1)^{st}$ iteration, calculate $P_i^{(r+1)}$ at all the PV and PQ buses and $Q_i^{(r+1)}$ at all the PQ buses, using voltages from previous iteration, $V_i^{(r)}$. The formulae to be used are

$$P_{i,cal} = P_i = G_{ii}|V_i|^2 + \sum_{k=1\&k\neq i}^{n} |V_1||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$Q_{i,cal} = Q_i = -B_{ii}|V_i|^2 + \sum_{k=1\&k\neq i}^{n} |V_1||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

4. Calculate the power mismatches (power residues)

$$\Delta P_i^r = P_{i,sp} - P_{i,cal}^{r+1}$$

$$(at\ PV\ and\ PQ\ buses)$$

$$\Delta Q_i^r = Q_{i,sp} - Q_{i,cal}^{r+1}$$

$$(at\ PQ\ buses)$$

5. Calculate the Jacobian [J (r)] using $V_i^{(r)}$ and its elements spread over H, N, M, L sub- matrices using the relations derived as in (36).

6. Compute

$$\begin{bmatrix} \Delta\delta^r \\ \dfrac{\Delta|V^{(r)}|}{|V|} \end{bmatrix} = \left[J^{(r)} \right]^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$$

7. Update the variables as follows:

$$\delta_i^{r+1} = \Delta\delta_i^r + \Delta\delta_i^r$$

$$(at\ all\ buses)$$

$$|V_i|^{r+1} = |V_i|^r + \Delta|V_i|^r$$

8. Go to step 3 and iterate till the power mismatches are within acceptable tolerance.

V. RESULTS AND DISCUSSIONS

The numerical analysis involving the solution of algebraic simultaneous equations forms the basis for solution of the performance equations in computer aided electrical power system analyses, such as during linear graph analysis, load flow analysis (nonlinear equations), transient stability studies (differential equations), etc. Hence, it is necessary to review the general forms of the various solution methods. In this work, the main objective is to determine the active and reactive power performance indices along with voltage and angle which form an important part of contingency analysis for different bus systems i.e. PIP and PIV respectively. The computation of these indices has been done based on load flow analysis carried out using Newton –Raphson and Gauss-Seidal methods under MATLAB environment. The study has been carried out for the following standard systems:

• 30 Bus system[16]

The IEEE 30 Bus Test Case represents a portion of the American Electric Power System (in the Midwestern US) as of December, 1961. The data was kindly provided by IrajDabbagchi of AEP and entered in IEEE Common Data Format by Rich Christie at the University of Washington in August 1993.

• 57 Bus system[15]

The IEEE 57 Bus Test Case represents a portion of the American Electric Power System (in the Midwestern US) as it was in the early 1960's. A hardcopy data was provided by Iraj Dabbagchi of AEP and entered in IEEE Common Data Format by Rich Christie at the University of Washington in August 1993. · This test case consists of 57 buses, 7 generators and 42 loads. The 57 bus test case does not have line limits.

• 118 Bus system [16]

Case 1: 30 bus system

The results of total loss for active and reactive power in the base case loading condition is obtained by using the newton-Raphson and Gauss-Seidal has been given in Table 5.1. along with time of convergence and number of iterations taken by both methods

Table 5.1: Output results for IEEE 30 bus system using Newton-Raphson method and Gauss elimination method

Iterations	Total Line Loss		Time	Time per iteration
Gauss-Seidal method- IEEE 30 BUS				
No.	MW	MVar	sec	Sec
270	17.512	69.547	0.1564	5.79E-04
Iterations	Total Line Loss		Time	Time per iteration
Newton-Raphson Method-IEEE 30 BUS				
No.	MW	MVar	sec	Sec
5	17.528	68.888	0.1809	0.0362

Case 2: 57 bus system

Similarly results for 57 IEEE bus systems has been given below

Table 5.2: Output results for IEEE 57 bus system using Newton-Raphson method and Gauss elimination method

Iterations	Total Line Loss		Time	Time per iteration
Gauss-Seidal method- IEEE 57 BUS				
No.	MW	MVar	sec	sec
467	55.206	346.83	0.4153	8.89E-04
Iterations	Total Line Loss		Time	Time per iteration
Newton-Raphson Method-IEEE 57 BUS				
No.	MW	MVar	sec	sec
11	22.703	169.196	0.6069	0.0552

Case 3: 118 bus system

Similarly results for 118 IEEE bus systems has been given below

Table 5.3: Output results for IEEE 118 bus system using Newton-Raphson method and Gauss elimination method

Iterations	Total Line Loss		Time	Time per iteration
Gauss-Seidal method- IEEE 118 BUS				
No.	MW	MVar	sec	sec
------Do not converge with tolerance 1e-6---- But converges if tolerance set more than .7				
Iterations	Total Line Loss		Time	Time per iteration
Newton-Raphson Method-IEEE 118 BUS				
No.	MW	MVar	sec	sec
12	285.468	1357.727	2.1435	0.1786

The comparison for both methods has been shown graphically in the figures below.

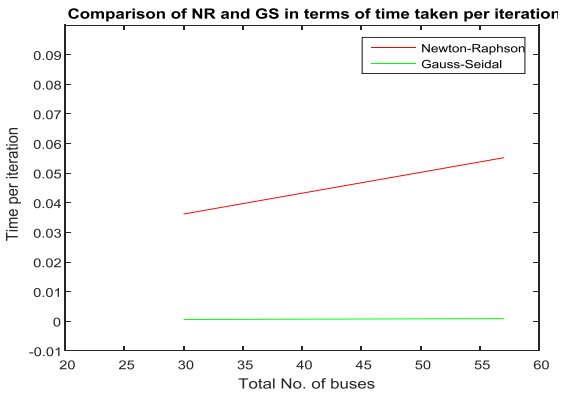

Figure 2: Time per Iteration in GS and NR methods

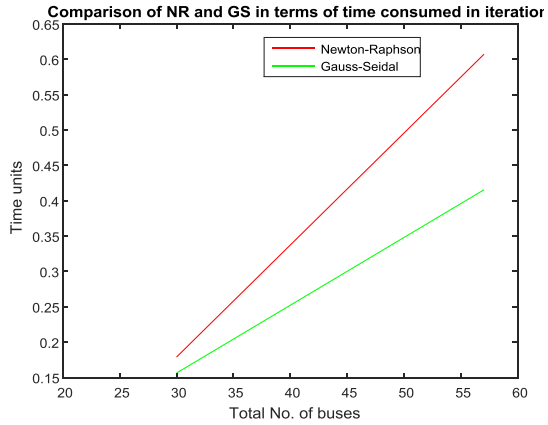

Figure 3: Total time of Iteration in GS and NR methods

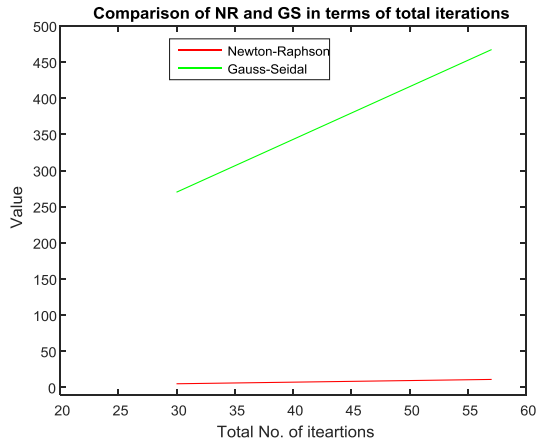

Figure 4: Total No. of iterations taken by NR and GS for different bus systems

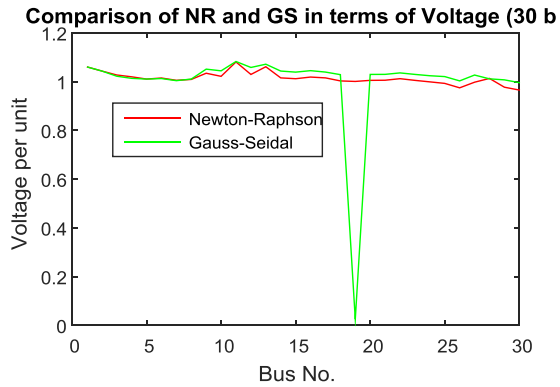

Figure 5: Comparison of NR and GS for 30 bus system in output voltage per unit evaluated at eachbus

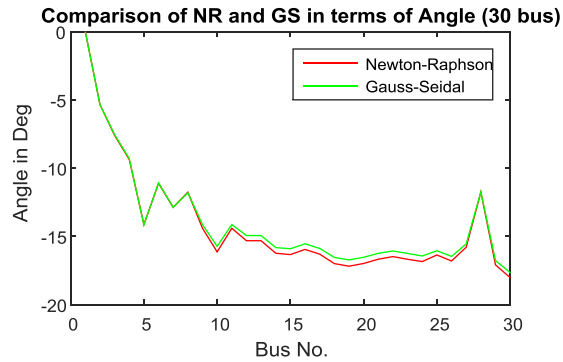

Figure 6: Comparison of NR and GS for 30 bus system in output angle in degree evaluated at eachbus

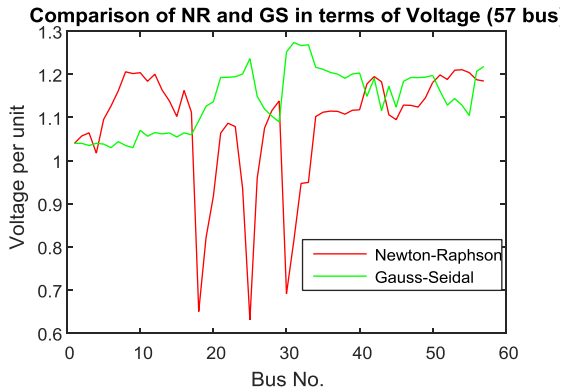

Figure 7: Comparison of NR and GS for 57 bus system in output voltage per unit evaluated at eachbus

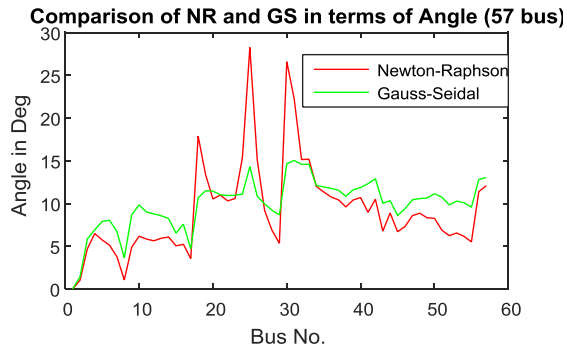

Figure 8: Comparison of NR and GS for 57 bus system in output angle in degree evaluated at eachbus

The convergence characteristic of the Newton–Raphson method is excellent. Generally, it can converge in 5-12 iterations, and the number of iteration does not depend on the scale of the power system. The Newton– Raphson method has a quadratic convergence characteristic if the initial guess values are close to the solution. If the initial guess values are not good enough, the iterative process may not converge or may converge to a solution at which the power system cannot operate. As described above, the substance of the Newton method is sequential linearization of nonlinear equations. Therefore, a good initial guess value is crucial because the Newton method is very sensitive to it. Under normal operation states of power systems, the node voltage magnitudes are usually close to their nominal voltages, and the phase angle differences between the nodes of a branch are not very large.

On the other hand, The GS method exhibits a linear convergence characteristic as compared to the NR method which has a quadratic convergence. Hence, the GS method requires more number of iterations to get a converged solution as compared to the NR method. In the GS method, the number of iterations increases directly as the size of the system increases. Also The Gauss –Seidel method requires the fewest number of operations to complete iteration. In the NR method, the computation of the Jacobian is necessary in every iteration. Further, the inverse of the Jacobian also has to be computed. Hence, the time per iteration is larger than in the GS method and is roughly about 7 times that of the GS method, in large systems, as depicted graphically in figure above. Computation time can be reduced if the Jacobian is updated once in two or three iterations. In this case study, Gauss-Seidal do not converge for 118 IEEE bus system with given tolerance of 1e-6. Then we tried it on different tolerances and it starts to converge when we put tolerance value of .7. Overall it has been evaluated that, Both methods give almost same results when there are few number of buses in the system and NR converge in fewer iterations as compared to that of Gauss-Seidal method. In terms of complexity, GS is less complex in equations. NR uses more memory for calculations as there are jacobian matrices in NR. It has been also found that NR is better in large quantity bus systems

VI. CONCLUSION

Speed, rate of convergence and the convergence characteristic of these iterative algorithms were confirmed by the case study results in the existed literature. Newton Raphson method is more reliable because it converges faster with quadratic convergence characteristics and least number of iterations when compared with the other two methods, In general the Newton Raphson algorithm takes the least number of iteration to converge despite its longer computing time. The number of iteration for the Gauss-Seidel increases directly as the number of the buses of the network, whereas the number of iterations for the Newton Raphson method remains practically constant, independent of the system size. However, since the convergence characteristics of the Fast decouple method is geometric compare to the quadratic convergence of the Newton Raphson, thus it has more number of iteration. Therefore because of high accuracies obtained in only a few iterations, the Newton Raphson method is important for use and more reliable than any of the methods.

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