Advance Physics Letter

Analysis of Functionally Graded Material Spur Gear under Static Loading Condition

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Abstract: Spur gear is widely used mechanical element in power transmission. The Gear tooth generally fails when applied load is increased above certain limit. The failure of gear tooth occurs due to pitting of gear tooth surface and / or bending stress developed at the root of gear tooth by the applied load during meshing of gear teeth. This can be avoided by using different materials for different applications. In order to increase the root strength of tooth and the life of the gear, functionally graded material (FGM) can be used for gear manufacturing .The modeling of a common cylindrical involute spur gear for both conventional and FGM has been done using SolidWorks2012 software. Then the static analysis of the performance of the subject under consideration is done using ANSYS PARAMETRIC DESIGN LANGUAGE 14.0 software for conventional gear materials (such as Cast Iron, Aluminium, Polycarbonate), Al-SiC metal matrix composite material and for a metal-ceramic functionally graded material .The finite element analysis of bending strength in terms of displacement of the gear models for various materials has been done. The results obtained in the different cases shows that gear teeth made of FGM has better performance for a specified load. The cylindrical involute spur gear tooth made of FGM experiences less displacement and strain developed by the application of same magnitude of tangential load on the tooth.

Index Terms—ANSYS Apdl, Exponential Law, FGM, Finite Element Analysis, Spur gear.

I. INTRODUCTION

Gear is a mechanical element, which plays an important role in power transmission system. Spur gears are very popular among all other types of gear due to its simplicity in design, less maintenance requirement and economic manufacturing. The gear fails when the applied load is increased above certain limits. There are two basic modes of gear tooth failure- breakage of tooth due to static and dynamic loads and surface destruction. The gear material should have sufficient strength to resist failure due to breakage of tooth [1].According to beam strength theory given by Lewis, spur gear tooth is treated as a cantilever beam to calculate the bending stress developed at root of thetooth. There are many factors which are considered while designing a gear like power transmitted, velocity ratio, material, gear tooth geometry (symmetric and/or asymmetric) and its various parameters like module, addendum, dedendum, face width ,fillet profile etc. To design a proper gear the following requirements must be fulfilled-(i) The teeth must be enough strong to withstand not only the static but dynamic load also, (ii) Wear resisting properties ensure the long life of gear, (iii) The teeth must be able to resist tooth deflection ,accelerations and stress concentration[2]. The above requirements shows that the material properties play the major role for sufficient strength gear design (beam strength as well as wear strength). Many research works have been accomplished using different types of materials for gear to increase its strength and other parameters. Also many researchers have proposed several concepts for gear design optimization to enhance the parameters of gear system. P.B.Pawar and Abhay A Utpat [3] in their work, compared metallic gears of steel alloy, Nylon gear and Aluminium Silicon carbide composite gear using finite element analysis in ANSYS and the comparison has been made and the result shows that the Composites provide much improved mechanical properties such as better strength to weight ratio, more hardness, and hence less chances of failure. Mrs. Shinde S.P. et al.[4] performed bending stress analysis , on a 3D deformable-body (model) of spur gears made of plain carbon steel .First, the finite element method and solution method for the accurate calculation of bending stresses has been determined, then bending stresses calculated using ANSYS, were compared to the results obtained from existing methods. According to the result, the numerically obtained values of stress distributions were in good agreement with the theoretical results.

The present study is based upon the investigation of the deflections, stress distributions and strains for the static response due to tangential load applied on a two dimensional single spur gear tooth for different materials using finite element analysis software ANSYS 14.0 Mechanical APDL. The simulations are carried out for same magnitude of load.

II. PROBLEM FORMULATION

i. Geometric data of Spur gear to be designed

In this study, spur gear has been designed for transmitting 1.5kW power at 1400 rpm according to Lewis theory

[1].The geometric parameters of spur gear are given as follows:

Module (m) =10mm

No. of teeth (Z) =18

Pressure angle (α) =20^0

Addendum =10mm

Dedendum =11.57mm

Pitch circle diameter (D_p) =180 mm

Tooth thickness (t) =15.71mm

Whole depth=22.5mm

Face width (b)=100mm

The torque and tangential force can be calculated as follows [1]:

Torque Transmitted, $M_t = \frac{60\times10^6\times kW}{2\pi N} = \frac{60\times10^6\times1.5}{2\pi\times1400}$

$$M_t = 10231.3892 \text{ N-mm}$$

Tangential force $F_t = \frac{2M_t}{D_p} = \frac{2\times10231.3892}{180}$

$$= 113.68 \text{ N}$$

ii. Mechanical Properties of various materials

Table-1: Mechanical Properties of Cast Iron, Aluminium [5] , Nylon, Polycarbonate [8] and Al-SiC Composite[3] and FGM [9]

Material	Young's Modulus of Elasticity E (N/mm^2)	Density ρ (kg/mm^3)	Poisson Ratio ν
Cast Iron	1.0×10^5	7.2×10^{-6}	0.23
Aluminium	0.675×10^5	1.13×10^{-6}	0.34
Al-Sic	1.5×10^5	2.9×10^{-6}	0.30
Poly-carbonate	2.75×10^3	1.1×10^{-6}	0.38
Steel-Zr FGM	$E_A=2.01\times10^5$ $E_B=2.44\times10^5$	$\rho_A=8.166\times10^{-6}$ $\rho_B=5.7\times10^{-6}$	$\nu_A=0.33$ $\nu_B=0.288$

iii. Theoretical Calculation of Bending Stress and Deflection

The theoretical value of maximum bending stress can be calculated using equation [5]

$$\sigma_{max} = \sigma_{nom} K_t \qquad \text{....... (1)}$$

Where,

$$\sigma_{nom} = \frac{6F_t h}{t^2} - \frac{F_t}{t}\tan\alpha \qquad \text{..... (1.a)}$$

Stress concentration factor [7]

$$K_t = 0.18 + \left[\left(\frac{t}{r}\right)^{0.15} + \left(\frac{t}{h}\right)^{0.45}\right] \qquad \text{..... (1.b)}$$

(For 20^0 full depth teeth system)

Maximum bending stress

$$\boldsymbol{\sigma_{max} = 2.10 N/mm^2}$$

Deflection for C.I. material

The deflection of the spur gear teeth is[6]

$$\delta_{max} = \frac{F_t h^3}{3EI} \qquad \text{...... (2)}$$

$$\boldsymbol{\delta_{max} = 2.89\times10^{-4} mm}$$

Similarly for considered materials

Table-2: Theoretical Calculation Results of Bending Stress and Deflection for different materials

Material	Force (F_t) N	Bending Stress (σ_{max}) N/mm^2	Deflection (δ) mm
Cast Iron	113.68	2.10	2.89×10^{-4}
Aluminium	113.68	2.10	4.28×10^{-4}
Al-Sic	113.68	2.10	1.93×10^{-4}
Poly-carbonate	113.68	2.10	1.0525×10^{-2}
Steel-Zr FGM	113.68	2.10	1.30×10^{-5}

iv. Functionally Graded Materials

Functionally Graded Material, (or sometimes also called'' functionally gradient material'') is characterized by a gradual change of material properties with dimension. In FGM, material function should be continuous or quasi-continuous. The examples of FGMs present in nature are bones, teeth etc., nature designed this materials to meet their expected service requirements[15-16]. Functionally graded material, eliminates the sharp interfaces existing in composite material which is where failure is initiated.

In this Analysis the material used for gear is a functionally graded material which follows the exponential law, given by the equation as following [14]:

$$E(r) = E_o e^{\beta r} \qquad \text{......(3)}$$

$$\rho(r) = \rho_o e^{\gamma r} \qquad \text{...... (4)}$$

$$\nu(r) = \nu_o e^{\mu r} \qquad \text{....... (5)}$$

Where,

$$E_o = E_A e^{-\beta a} \qquad \text{...... (3.a)}$$

$$\rho_o = \rho_A e^{-\gamma a} \qquad \text{...... (4.a)}$$

$$\nu_o = \nu_A e^{-\mu a} \qquad \text{...... (5.a)}$$

$$\beta = \frac{1}{(a-b)}\ln\left(\frac{E_A}{E_B}\right) \qquad \text{...... (3.b)}$$

$$\gamma = \frac{1}{(a-b)}\ln\left(\frac{\rho_A}{\rho_B}\right) \qquad \text{...... (4.b)}$$

$$\mu = \frac{1}{(a-b)} \ln\left(\frac{\nu_A}{\nu_B}\right) \qquad \text{...... (5.b)}$$

a = inner radius

b = outer radius

E_A= Young’s Modulus of elasticity for Steel

E_B= Young’s Modulus of elasticity for Zirconia (Zr)

ρ_A= Density of steel

ρ_B= Density of Zr

The inner surface(r=a) of the spur gear tooth consists of full metal A (Steel) and the outer surface (r = b) of the gear tooth consists of fullceramic material B (Zr).For Theoretical analysis the effective Properties of FGM is evaluated using Mori-Tanaka Scheme[10]:

$$\frac{K-K_A}{K_B-K_A} = \frac{V_B}{1+\frac{3(1-V_B)(K_B-K_A)}{3K_A+4G_A}} \qquad \text{...... (6)}$$

$$\frac{G-G_A}{G_B-G_A} = \frac{V_B}{1+\frac{(1-V_B)(G_B-G_A)}{G_A+f_A}} \qquad \text{...... (7)}$$

Where, $f_A = \frac{G_A(9K_A+8G_A)}{6(K_A+2G_A)}$ ...... (7.a)

Volume fraction $V_A + V_B = 1$ .... (8)

Where,

$$K_A = \frac{E_A}{2(1+\nu_A)} \qquad \text{...... (6.a)}$$

$$K_B = \frac{E_B}{2(1+\nu_B)} \qquad \text{...... (6.b)}$$

$$G_A = \frac{E_A}{3(1-2\nu_A)} \qquad \text{..... (7.b)}$$

$$G_B = \frac{E_B}{3(1-2\nu_B)} \qquad \text{..... (7.c)}$$

Where K and G are bulk modulus and modulus of rigidity of the considered materials respectively.

Hence the effective Young’s Modulus of Elasticity of FGM is evaluated by:-

$$E = \frac{9KG}{3K+G} \qquad \text{...... (9)}$$

And density is given by:

$$\rho = \rho_A V_A + \rho_B V_B \qquad \text{....... (10)}$$

and Poisson’s ratio is evaluated by taking average of the two values of FGM constituents.

III. METHODOLOGY

Based on the above literature reviews it has been found that the previous research in this field was basically concentrated on composite materials as well as conventional materials.

From the literature review it has been observed that functionally graded materials can be used for gears. So on this basis this research is carried out using functionally graded material to analyze the gear tooth. The methodology of the complete research is presented by the process:

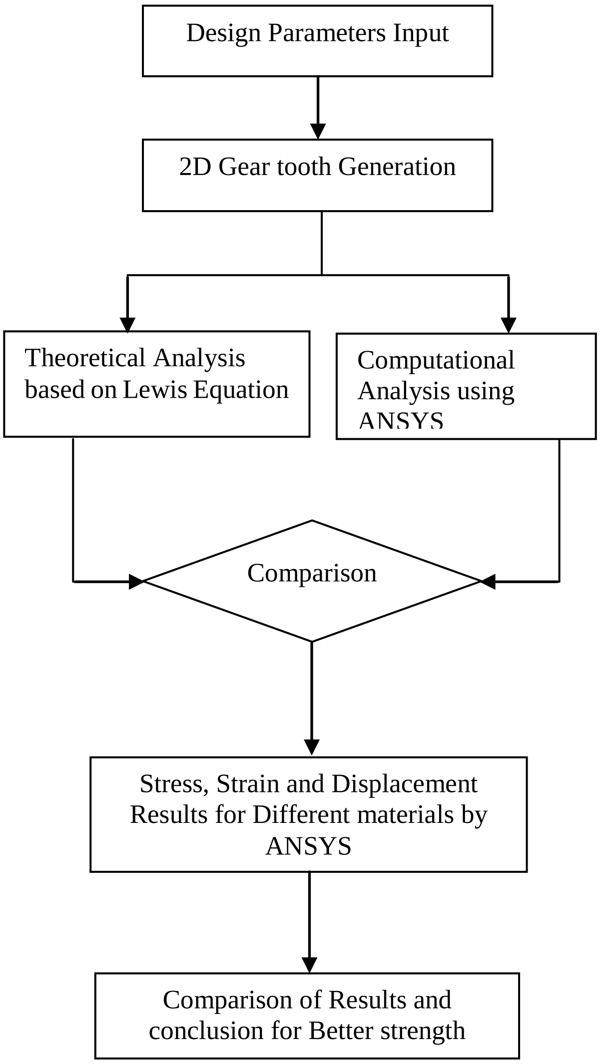

Fig. 1.Process flow of Methodology

IV. COMPUTATIONAL ANALYSIS USING ANSYS APDL 14.0

i. Finite Element Analysis

The finite element analysis is performed using ANSYS Mechanical APDL 14.0. The plane stress analysis of a two dimensional single spur gear tooth is carried out treating the tooth as a cantilever beam to evaluate the strength of the tooth in terms of strain and displacement.

ii. Involute profile Generation of Gear tooth

The gear tooth involute profile is generated by using SolidWorks2012 and also the key points are generated by dividing it in various sectors. These key points are used to create the parametric model of spur gear tooth in ANSYS parametric design language software.

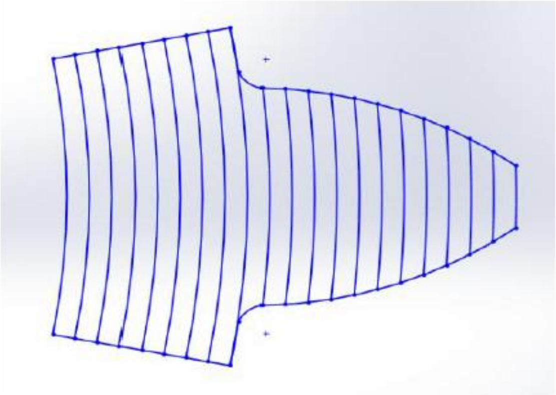

Fig.2. Involute gear tooth creation in Solid Works

Fig.3.Parametric modeling of gear tooth in ANSYS

iii. Two Dimensional Finite Element Analysis

In this research a 2D gear tooth is analyzed by treating it as a cantilever beam. Basically the plane stress analysis is done to evaluate the results. In plane stress analysis the normal stress σ_z , and the shear stresses σ_{xz} ,and σ_{yz} directedperpendicular to the x-y plane are assumed to be zero [6].

The tangential force is applied uniformly over the thickness of the tooth plane, same as applied in the cantilever beam when plane stress analysis is to be performed.

For the considered analysis the material properties are defined for the gear tooth. The analysis is carried out by constraining the inner rim of the gear tooth and the two radial lines are constrained in both the directions .Then the force of magnitude 113.68 N is tangentially applied at the tip of the gear tooth as per Lewis theory.

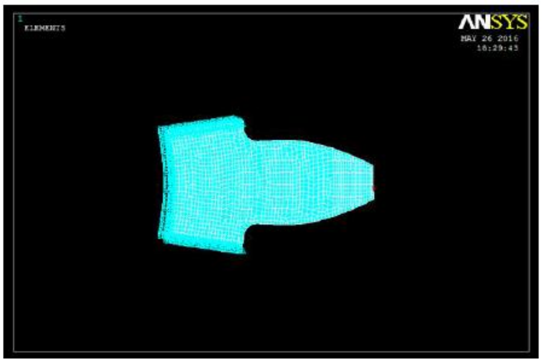

Fig.4. BC’s and Forceapplied

The various results obtained in the ANSYS for the functionally graded material gear tooth are given in the figures below-

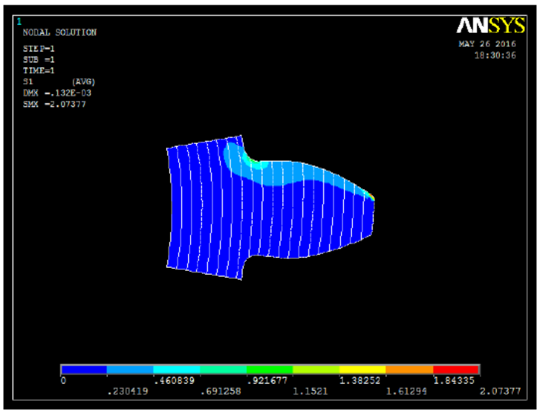

Fig.5. Bending Stress

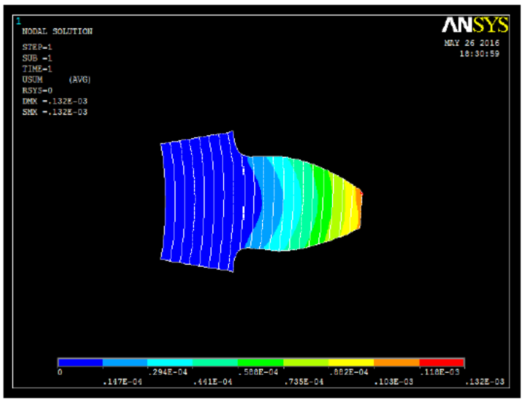

Fig.6. Deflection

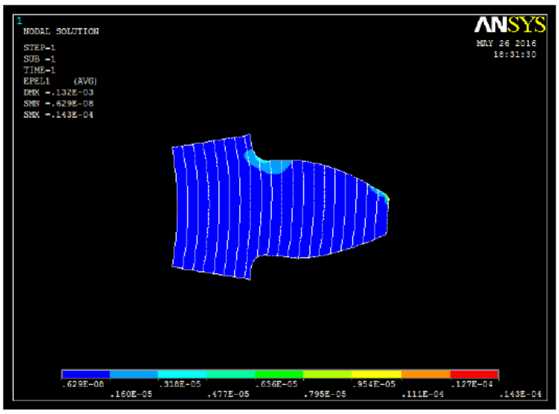

Fig.7. Strain

Results obtained in ANSYS for the considered material are tabulated below:

Table-3: ANSYS Finite Element Analysis Results

Material	**Force N**	**Bending Stress (σ_{max}) N/mm2**	**Deflection (δ) (mm)**	**Strain Ɛ**
Cast Iron	113.68	2.04611	2.97×10^{-4}	3.26×10^{-5}
Aluminium	113.68	2.04429	4.41×10^{-4}	5.7×10^{-5}
Al-Sic	113.68	2.04477	1.98×10^{-4}	2.42×10^{-5}
Poly-carbonate	113.68	2.04405	1.0831×10^{-2}	1.477×10^{-3}
Steel-Zr FGM	113.68	2.07377	1.32×10^{-5}	1.43×10^{-5}

V. RESULTS AND DISCUSSION

Table-4: Computational results of bending stress in ANSYS for various materials are as follows:

Material	**Theoretical (σ_{max}) N/mm2**	**Computational (σ_{max}) N/mm2**	**Error (%)**
Cast Iron	2.1	2.04611	2.56
Aluminium	2.1	2.04429	2.65
Al-Sic	2.1	2.04477	2.63
Poly-carbonate	2.1	2.04405	2.66
Steel-Zr FGM	2.1	2.07377	1.25

Table-5: Computational results of deflection in ANSYS for various materials are as follows:

Material	**Theoretical Deflection (δ)mm**	**Computational Deflection (δ)mm**	**Error (%)**
Cast Iron	2.89×10^{-4}	2.97×10^{-4}	2.69
Aluminium	4.28×10^{-4}	4.41×10^{-4}	2.94
Al-Sic	1.93×10^{-4}	1.98×10^{-4}	2.52
Poly-carbonate	1.0525×10^{-2}	1.0831×10^{-2}	2.82
Steel-Zr FGM	1.30×10^{-5}	1.32×10^{-5}	1.54

Discussion-

The static analysis has been performed for the different materials to be used for manufacturing spur gear. By this analysis it was observed that the bending stress value was increased for the FGM gear but it experiences less deflectionand strain as compared to the considered conventional materials (CastIron, Aluminium, and Poly-carbonate) and Aluminium-Silicon Carbide (Al-SiC) composite material. As per the study the Steel-Zirconium functionally gradient material can be used to manufacture spur gear for better strength, less distortion and greaterload carrying capacity.

VI. CONCLUSION

Based on the analysis it has been observed that the difference between the values of theoretical and computational bending stresses developed in spur gear tooth material are from 1.25% to 2.66%.

The study result infers the difference between theoretical and computational value of deflection from 1.54% to 2.94%.

Bending stress, strain and deflection occurred in the spur gear tooth due to the tangential load depend on the material property in finite element analysis.

The minimum strain value was observed for FGM (1.43×10^{-5}) by computational analysis.

The minimum deflection value was observed for FGM(1.32×10^{-5}mm) by computational analysis.

The observations by the study and analysis concluded that the functionally graded materials can be used for spur gear manufacturing for better strength and load carrying capacity.

Functionally Graded material modeled using power law can be further analyzed for different gears.

Various Functionally graded materials can be applied instead of currently used material.

Gears of any material having circular root fillet design can be investigated using similar procedure. It is important because it is observed that use of circular root fillet increases the beam strength of gear tooth as compare to that in case of trochoidal root fillet.

A study on wear, friction and temperature effects can be extended.

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